14.6/14.7 Homework

14.6

1. a. $\frac{\partial f}{\partial x} = 2xy^3$ $\quad \frac{\partial f}{\partial y} = 3x^2y^3$ $\quad \frac{\partial f}{\partial z} = 4z^3$

b. $\frac{\partial x}{\partial s} = 2s$ $\quad \frac{\partial y}{\partial s} = t^2$ $\quad \frac{\partial z}{\partial s} = 2st$

c. $\frac{\partial f}{\partial s} = (2xy^3)(2s) + (3x^2y^3)(t^2) + (4z^3)(2st)$

$= 4xy^3s + 3x^2y^3t^2 + 8z^3st$

$= 7s^6t^6 + 8s^7t^4$

3. $\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{dx}{ds} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial s}$

$= (y)(2s) + (x)(2r) + (2z)(0)$

$= 4rs^2 + 2s^2r$ $\boxed{= 6rs^2}$

$\frac{\partial f}{\partial r} = (y)(0) + (x)(2s) + (2z)(2r)$

$\boxed{= 2s^3 + 4r^3}$

5. $\frac{\partial g}{\partial u} = -\sin(x-y)(3) + \sin(x-y)(-7)$

$\boxed{= 10\sin(10u - 20v)}$

$\frac{\partial g}{\partial v} = -\sin(x-y)(-5) + \sin(x-y)(15)$

$\boxed{= 20\sin(10u - 20v)}$

7.

$\frac{\partial F}{\partial y} = \frac{\partial F}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial F}{\partial v}\frac{\partial v}{\partial y}$

$= (e^{u+v})(0) + (e^{u+v})(x)$

$= e^{u+v}(x)$

$\boxed{= xe^{x^2+xy}}$

15. $\frac{\partial g}{\partial u} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial u}$

$= (2x)(e^u\cos v) + (-2y)(e^u\sin v)$

$2e^{2u}\cos^2 v - 2e^{2u}\sin^2 v$

$2\cos^2 1 - 2\sin^2 1 \; \boxed{= -.83229}$

23. $\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}$

$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$

$\frac{\partial f}{\partial s}\frac{\partial f}{\partial t} = \left[\frac{\partial f}{\partial x}(1) + \frac{\partial f}{\partial y}(-1)\right]\left[\quad\right]$

$\left[\frac{\partial f}{\partial x}(1) + (-1)\frac{\partial f}{\partial y}\right]$ Difference of squares

$$\boxed{= \left[\frac{\partial f}{\partial x}\right]^2 - \left[\frac{\partial f}{\partial y}\right]^2}$$

27. $0 = 2xy + y^2\frac{\partial z}{\partial x} + 2xz\frac{\partial z}{\partial x} + z^2$

$$\boxed{\frac{\partial z}{\partial x} = \frac{-2xy - z^3}{y^2 - 2xz}}$$

29.

$$0 = xe^{xy} + x\frac{\partial z}{\partial y}\cos(xz) + 1$$

$$\boxed{\frac{-1 - xe^{xy}}{x\cos(xz)} = \frac{\partial z}{\partial y}}$$

31.

$$1 = (w^2+x^2)^{-1} + (w^2+y^2)^{-1}$$

$$0 = \frac{-2w}{(w^2+x^2)^2}\frac{\partial w}{\partial y} - \frac{2w+2y}{(w^2+y^2)^2}\frac{\partial w}{\partial y}$$

$$\frac{2w+2y}{(w^2+y^2)^2} + \frac{2w}{(w^2+x^2)^2} = \frac{\partial w}{\partial y}$$

$$\left.\frac{\partial w}{\partial y}\right|_{(x,y,w)=1,1,1} \boxed{= 1.8}$$

14.7

1. a $f_x(x,y) = 2x - 4y = 0$

$x = 2y$

$a = 2b$

$f_y(x,y) = 4y^3 - 4x = 0$

$x = y^3$ $\quad 2y^3 - 4y = 0$

$(2\sqrt{2}, \sqrt{2}), (-2\sqrt{2}, -\sqrt{2})$ $\quad 2y(y^2 - 2) = 0$

$y = \pm\sqrt{2}$

$x = \pm 2\sqrt{2}$

b. $f_x = 2x - 4y$
$f_y = 4y^3 - 4x$
$f_{xx} = 2$
$f_{yy} = 12y^2$
$f_{xy} = -4$

critical Points:

$(2\sqrt{2}, \sqrt{2})$ $(0,0)$

$(-2\sqrt{2}, -\sqrt{2})$

$D_{(2\sqrt{2}, \sqrt{2})} = 32$ $D_{(-2\sqrt{2}, -\sqrt{2})} = 32$

$f_{xx} = 2$ $f_{xx} = 2$

$D_{(0,0)} = -16$ $(0,0)$ = saddle point

$(2\sqrt{2}, \sqrt{2})$, $(-2\sqrt{2}, -\sqrt{2})$ are local minima

3. $f_x = 2x + y = 0$ $f_y = 32y^3 + x - 6y - 8y^2 = 0$

$-x = y/2$

$32y^3 - 8y^2 - 13/2\,y = 0$

$y = \frac{-13}{32}, \frac{1}{2}, 0$

$(0,0)$ $(13/64, -13/32)$ $(-1/4, \frac{1}{2})$

SADDLE

LOCAL MINIMA

$f_{xx} = 2$
$f_{yy} = 96y^2 - 6 - 6y$
$f_{xy} = 0$

5. a. $f_x = y^2 - 2xy + y = y(y-2x+1)$ ✓

$0 = \begin{cases} f_x \\ f_y = 2xy - x^2 + x = x(2y - x + 1) \end{cases}$ ✓

b. $(0,0)$ $(1,0)$ $(0,-1)$ $(1/3, -1/3)$

c. SADDLE POINTS local min

$f_{xx} = -2y$ $f_{yy} = 2x$ $f_{xy} = 2y - 2x + 1$

7. $f_x = 2x - y + 1$ $f_y = 2y - x$ $2y = x$

$f_{xx} = 2$ $f_{yy} = 2$

$f_{xy} = -1$

D always equals 3 $f_{xx} f_{xy} - [f_{xy}]^2$

$y = -1/3$ $x = -2/3$ $(-2/3, -1/3)$ Local min

11. $f_x = 4 - 9x^2 - 2y^2$ $f_y = -4xy$ x or y = 0

$f_{xx} = -18x$ $f_{yy} = -4x$

f_{xy} 0

$D = 72x^2$ $(0, \sqrt{2})$ $(0, -\sqrt{2})$

Local min: $(0, \sqrt{2}), (0 - \sqrt{2})$ $\left(\frac{2}{3}, 0\right)$ $\left(\frac{-2}{3}, 0\right)$

$(2/3, 0)$

Local max: $(2/3, 0)$

13. $f_x = 4x^3 - 4y$ $\quad$ $f_y = 4y^3 - 4x$

$f_{xx} = 12x^2$ $\quad$ $f_{yy} = 12y^2$

$f_{xy} = -4$

$D = 144x^2y^2 - 16$

$x^3 = y$ $\quad$ $y^3 = x$ $\quad$ $x, y = \pm 1$

$(1,1)$ $\quad$ $(-1,-1)$ $\quad$ $(0,0)$

$D_{(1,1)} = 128$ $\quad$ $(1,1)$ Local min

$D_{(-1,-1)} = 128$ $\quad$ $(-1,-1)$ Local min

$D_{(0,0)} = -16$ $\quad$ $(0,0)$ Saddle point

14. $f_x = \frac{1}{x} - 1$ $\quad$ $f_y = \frac{2}{y} - 4$

$f_{xx} = \frac{-1}{x^2}$ $\quad$ $f_{yy} = \frac{-2}{y^2}$

$f_{xy} = 0$ $\quad$ $x = 1$

$y = 1/2$

$D = \frac{2}{x^2y^2}$

$f_{xx} = -1$

$D_{(1, 1/2)} = >0$

$(1, 1/2)$ Local max

21. $f_x = 1 - \frac{1}{x+y}$ $f_y = -2y - \frac{1}{x+y}$

$f_{xx} = \frac{1}{(x+y)^2}$ $f_{yy} = -2 + \frac{1}{(x+y)^2}$

$f_{xy} = \frac{1}{(x+y)^2}$ $x+y=1$ $(3/2, -1/2)$

$x+y = \frac{-1}{2y}$

$D < 0$, $(3/2, -1/2)$ SADDLE POINT

$D = -1 - 1 = -2$

23. $f_x = (x+3y) - 2x e^{y-x^2} + e^{y-x^2}$

$f_y = (x+3y) e^{y-x^2} + 3 e^{y-x^2}$

$f_{xx} = 4e^{y-x^2} x^3 + 12 e^{y-x^2} y x^2 - 6e^{y-x^2} x - 6e^{y-x^2} y$

$f_{yy} = -6ye^{y-x^2} + e^{y-x^2} - 2e^{y-x^2} x^2 - 6e^{y-x^2} x$

$f_{xy} = -6x(e^{y-x^2} + e^{y-x^2} y) - 2x^2 e^{y-x^2} + e^{y-x^2}$

IM NOT SURE HOW TO

SOLVE THIS

29. Min: 0 $(0,0)$ Direct Substitution

Max: 2 $(1,1)$

35. $f_x = 1 - 2x - y$ $f_y = 1 - 2y - x$ $-x = y$

$f_{xx} = -2$ | Local min $(-1,1)$ $(1,1)$ | $f_{yy} = -2$

$f_{xy} = -1$

D always > 0 f_{xx} always < 0