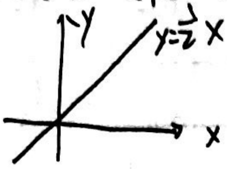
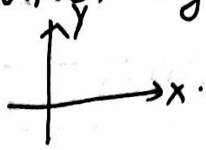
Exercise 15.6

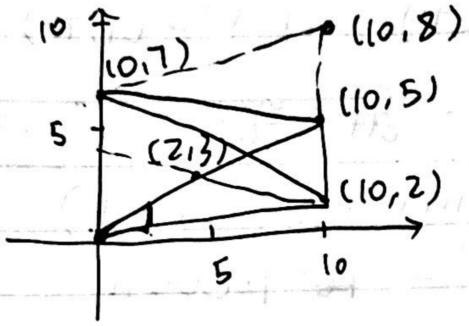
Q1. $\because y = \frac{1}{2}x$ (for u) for v axis, is y-axis.



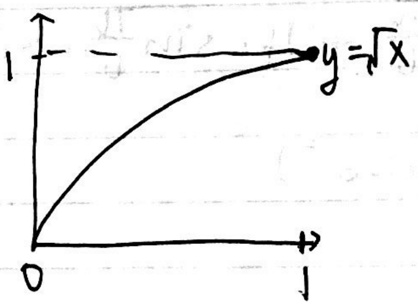


the parallelogram with verticles (0,0) (10,5) (10,2) (0,7)

$x = 0..5$ $y = 0..7$



Q3. G is not one to one, the domain for one to one is ~~$u \geq 0$~~ $\{(u,v) \mid u \geq 0\}$ $\{(u,v) \mid u \leq 0\}$.



No.

Date

$$\frac{dx}{du} \quad \frac{dx}{dv}$$
$$\frac{dy}{du} \quad \frac{dy}{dv}$$

Q13. $G(u,v) = (3u+4v, u-2v)$

$$\text{Jacobian } G = \frac{dx}{du} \cdot \frac{dy}{dv} - \frac{dx}{dv} \cdot \frac{dy}{du}$$
$$= 3 \cdot (-2) - 4 \cdot 1$$
$$= -6 - 4 = -10$$

Q15. $G(r,t) = (r\sin t, r - \cos t)$ $\quad (r,t) = (1, \pi)$

$$\text{Jacobian } G = \frac{dx}{dr} \cdot \frac{dy}{dt} - \frac{dx}{dt} \cdot \frac{dy}{dr}$$
$$= \sin t \cdot \sin t - (r\cos t \cdot 1)$$
$$= (\sin \pi)^2 - \cancel{\quad} 1 \cdot \cos \pi$$
$$= 0 - (-1) = 1$$

Q17. $G(r,\theta) = (r\cos\theta, r\sin\theta)$, $(r,\theta) = (4, \frac{\pi}{6})$

$$\text{Jacobian } G = \frac{dx}{dr} \cdot \frac{dy}{d\theta} - \frac{dx}{d\theta} \cdot \frac{dy}{dr}$$
$$= \cos\theta \cdot r\cos\theta + r\sin\theta \cdot \sin\theta$$
$$= \cos\frac{\pi}{6} \cdot 4 \cdot \cos\frac{\pi}{6} + 4 \cdot \sin\frac{\pi}{6} \cdot \sin\frac{\pi}{6}$$
$$= 4(\sin\theta^2 + \cos\theta^2)$$
$$= 4$$

Campus

No.

Date

Q19. $[0,1]\times[0,1]$ $\quad (2,3)\ (4,1)$ $\quad G(x,y)=(ax+by,\ cx+dy)$

$\therefore\ G(0,0)=(0,0)$ $\quad G(1,0)=(a,c)$

$G(1,0)=\cancel{(2,3)}\ (4,1)$ $\quad G(0,1)=(b,d)$

$G(0,1)=\cancel{(4,1)}\ (2,3)$ $\therefore\ a=4\ \ b=2\ \ c=1\ \ d=3$

$G(x,y)=(4x+2y,\ x+3y).$

$G(u,v)=(4u+2v,\ u+3v)$

Q23. $G(u,v)=(3u+v,\ u-2v)$ $\quad$ (a). $R=[0,3]\times[0,5]$

(a): Area $=3\times5=15$

Jacobian $=\begin{vmatrix}3 & 1\\ 1 & -2\end{vmatrix}=-6-1=-7$

$|-7|\times15=|105|=105$

the area $G(R)=105.$

(b) $R=[2,5]\times[1,7]$

Jacobian $=-7$

$\int_2^5\int_1^7 (-7)\, dy\, dx=-126$

$|-126|=126$

$\therefore$ area of $G(R)=126$

No.

Date

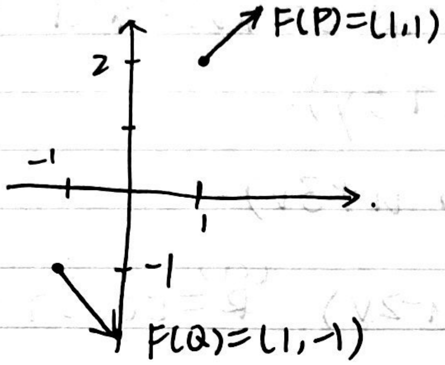
Exercise 16.1

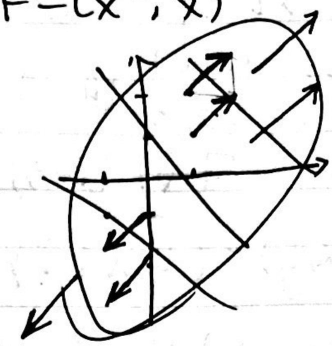
Q:

Q1. $P=(1,2)$ $Q=(-1,-1)$ $F=(x^2,x)$

$P(1,2)$ $F=(1,1)$

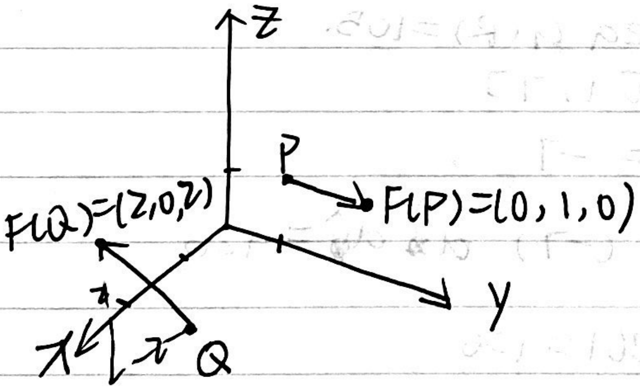
$Q(-1,-1)$ $F=(1,-1)$



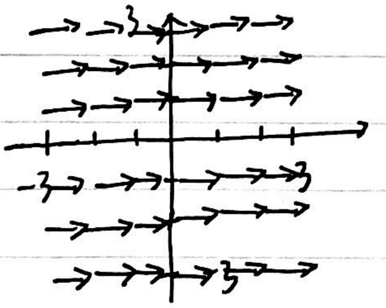


Q3. $P=(0,1,1)$ $Q=(2,1,0)$ $F=(xy, z^2, x)$

$F(P)=(0,1,0)$ $F(Q)=(2,0,2)$



Q5. $-3 \le x \le 3$, $-3 \le y \le 3$ $F=(1,0)$



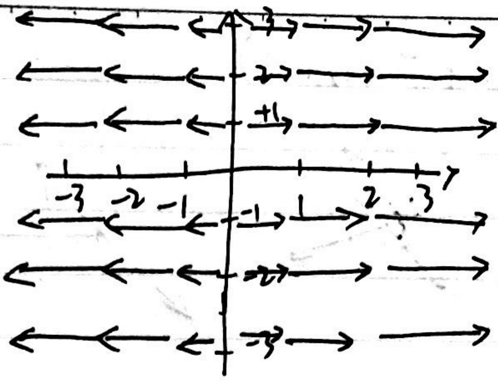
Q7. $F = x\mathbf{i}$.

$-3 \le x \le 3, \ -3 \le y \le 3$.

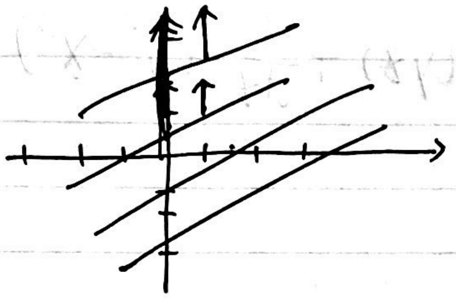
$(0,0) \quad F=(0,0)$

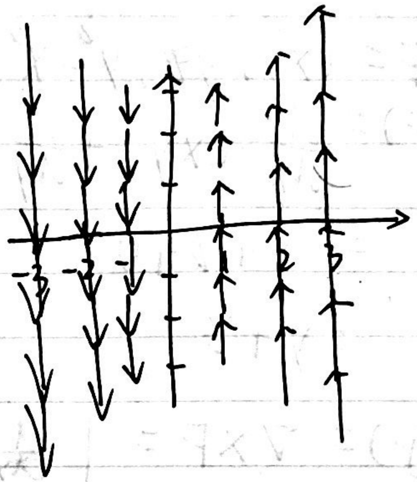
$(1,0) \quad F=(1,0)$

$(2,0) \quad F=(2,0)$

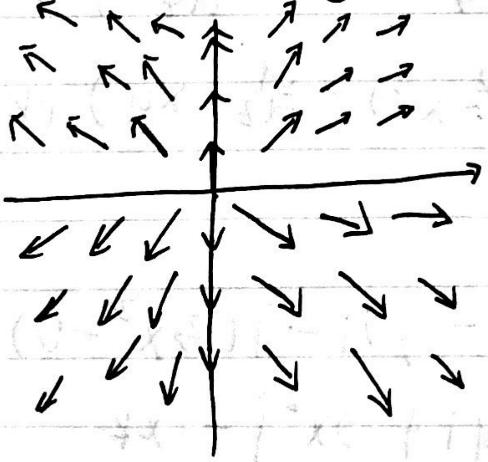


Q9. $F=(0,x)$

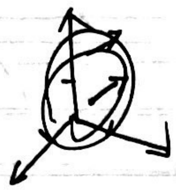


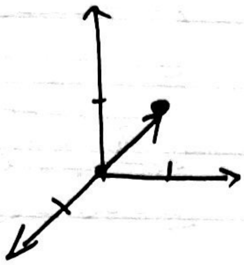


Q11. $F=\left(\dfrac{x}{x^2+y^2}, \dfrac{y}{x^2+y^2}\right)$



Q17. $F=(1,1,1)$





$\therefore$ plot(D)

Q23. $F=(xy, yz, y^2-x^3)$

$$\operatorname{div}(F)=\frac{d}{dx}\cdot(xy)+\frac{d}{dy}\cdot(yz)+\frac{d}{dz}\cdot(y^2-x^3)$$

$$= y+z+0$$

$$=y+z$$

$$\operatorname{curl}(F)=\nabla\times F=\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ xy & yz & y^2-x^3 \end{vmatrix}$$

$$=\left(\frac{d}{dy}\cdot(y^2-x^3)-\frac{d}{dz}\cdot yz\right)i-j\left(\frac{d}{dx}\cdot(y^2-x^3)-\frac{d}{dz}\cdot xy\right)+k\left(\frac{d}{dx}\cdot yz-\frac{d}{dy}\cdot xy\right)$$

$$=(2y-y)i-j(3x^2-0)+k(0-x)$$

$$= yi+3x^2j-xk$$

$$=(y,+3x^2,-x)$$

No.

Date

Q25. $F = (x - 2zx^2,\ z - xy,\ z^2x^2)$

$\operatorname{div}(F) = (1 - 4xz,\ -x,\ 2zx^2)$

$= 1 - 4xz - x + 2x^2 z$

$\operatorname{curl}(F) = \cancel{a}\ i(0-1) - j(2xz^2 - (-2x^2)) + k(-y-0)$

$= (-1,\ 2x^2 - 2xz^2,\ -y)$

Q27. $F = (z - y^2,\ x + z^3,\ y + x^2)$

$\operatorname{div}(F) = (0, 0, 0) = 0$

$\operatorname{curl}(F) = i(1 - 3z^2) - j(2x - 1) + k(1 + 2y)$

$= (1 - 3z^2\cancel{x},\ 1 - 2x,\ 1 + 2y)$