

15.6

1.

(a) u axis: $y = \frac{1}{2}x$
v-axis: $y = -x$

(b) the parallelogram with vertices $(0,0)$ $(0,1)$ $(1,2)$ $(1,1)$

(c) the segment joining the points $(2,3)$ and $(1,1)$

(d) the triangle with vertices $(0,1)$ $(2,1)$ and $(2,2)$.

3.

(a) positive x-axis including the origin and y-axis.

(b) the ~~area~~ rectangle $(0,1) \times (-1,1)$

(c) the curve $y = \sqrt{x}$ for $0 \leq x \leq 1$

(d) ..

13. $C(u,v) = (3u+4v, u-2v)$

$$x = 3u + 4v \quad y = u - 2v$$

$$\frac{dx}{du} = 3 \quad \frac{dx}{dv} = 4$$

$$\frac{dy}{du} = 1 \quad \frac{dy}{dv} = -2$$

$$3 \times (-2) - 1 \times 4 = -6 - 4 = -10$$

17. $C(r,\theta) = (r \cos \theta, r \sin \theta)$ $(r, \theta) = (4, \frac{\pi}{6})$

$$x = r \cos \theta \quad y = r \sin \theta$$

$$\frac{dx}{dr} = \cos \theta = \frac{\sqrt{3}}{2} \quad \frac{dx}{d\theta} = -r \sin \theta = -2$$

$$\frac{dy}{dr} = \sin \theta = \frac{1}{2} \quad \frac{dy}{d\theta} = r \cos \theta = 2\sqrt{3}$$

$$\frac{\sqrt{3}}{2} \times 2\sqrt{3} - \frac{1}{2} \times (-2) = 3 + 1 = 4$$

15. $C(t) = (r \sin t, r \cos t)$
 $x = r \sin t \quad y = r \cos t \quad (r, t) = (1, \pi)$

$$\frac{dx}{dt} = r \cos t = 0 \quad \frac{dy}{dt} = -r \sin t = -1$$

$$\frac{dy}{dx} = -1 \quad \frac{dy}{dt} = \sin t = 0$$

$$0 \times 0 - 1 \times (-1) = 1$$

19. $(0,1) = (2,3)$

$(0,1) = (4,1)$

$C(u,v) = (Au+Bv, Cu+Dv)$

$C(0,1) = (A, C) = (2,3) \Rightarrow A=2, C=3$

$C(0,1) = (A, C) = (4,1) \Rightarrow A=4, C=1$

$$B=4, D=1$$

$C(u,v) = (4u+4v, u+2v)$



23.

$$\gamma(u, v) = (3u + v, u - 2v)$$

$$a) R = [0, 3] \times [0, 1.5]$$

$$x = 3u + v \quad y = u - 2v$$

$$\frac{dy}{du} = 1 \quad \frac{dy}{dv} = -2$$

$$\frac{dy}{du} = 1 \quad \frac{dy}{dv} = -2$$

$$3 + (-2) - 1 = \frac{3-2-1}{-6-1} = -7$$

$$(3-0) \times (5-0) \times |-7| = 15 \times 7 = 105$$

$$b) R = [2, 5] \times [1, 7]$$

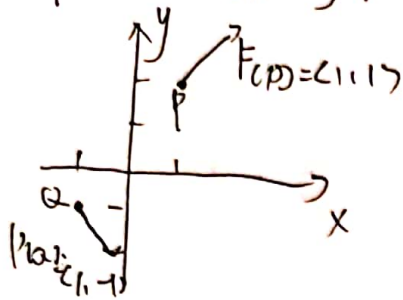
$$(5-2) \times (7-1) \times |-7| = 126$$



16.1

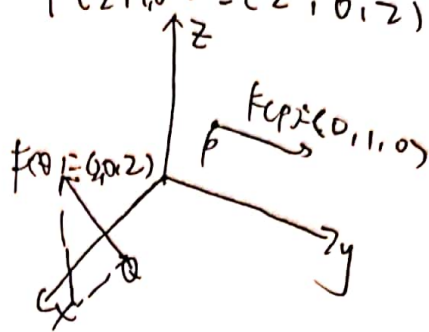
1. $F(1,2) = (1,1)$

$F(-1,-1) = (1,-1)$

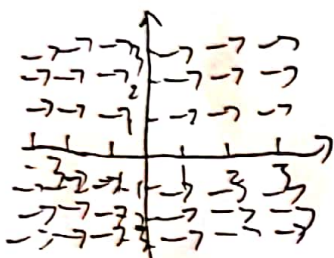


3. $F(0,1,1) = (0,1,0)$

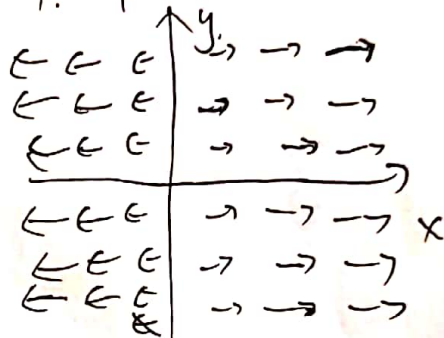
$F(2,1,0) = (2,0,2)$



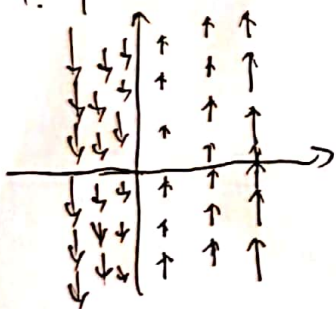
5. $F = (1,0)$



7. $F = xi$



9. $F = (0, x)$



11. $F = \left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right)$



17. $F = (4, 1, 1)$

plot (c)

23. $F = (xy, yz, y^2 - x^3)$

i	j	k
$\frac{d}{dx}$	$\frac{d}{dy}$	$\frac{d}{dz}$
xy	yz	$y^2 - x^3$

$$i \left(\frac{d}{dy} (yz) - \frac{d}{dz} (yz) \right) - j \left(\frac{d}{dx} (y^2 - x^3) - \frac{d}{dz} (xy) \right)$$

$$+ k \left(\frac{d}{dx} (yz) - \frac{d}{dy} (xy) \right) = yi + 3x^2j - xk = (y, 3x^2, -x)$$



$$\text{div}(F) = \frac{d}{dx} xy + \frac{d}{dy} yz + \frac{d}{dz} xy^2 - x^3$$

$$= y + z$$

$$\text{v) } F(x - 2zx^2, z - xy, z^2x^2)$$

i	j	k
$\frac{d}{dx}$	$\frac{d}{dy}$	$\frac{d}{dz}$
$x - 2zx^2$	$z - xy$	z^2x^2

$$i \left(\frac{d}{dy} (z^2x^2) - \frac{d}{dz} (z - xy) \right) - j \left(\frac{d}{dx} (z^2x^2) - \frac{d}{dz} (x - 2zx^2) \right)$$

$$+ k \left(\frac{d}{dx} (z^2x^2) - \frac{d}{dy} (z - xy) \right)$$

$$= -i - j(2xz^2 - x^2) - kyz$$

$$= (-1, 2xz^2 - x^2, -yz)$$

$$\text{div}(F) = \frac{d}{dx} (x - 2zx^2) + \frac{d}{dy} (z - xy) + \frac{d}{dz} (z^2x^2)$$

$$= 1 - 4xz - x + 2x^2z$$

$$\text{v) } F = (z - y^2, x + z^3, y + x^2)$$

i	j	k
$\frac{d}{dx}$	$\frac{d}{dy}$	$\frac{d}{dz}$
$z - y^2$	$x + z^3$	$y + x^2$

$$i \left(\frac{d}{dy} (y + x^2) - \frac{d}{dz} (x + z^3) \right) - j \left(\frac{d}{dx} (y + x^2) - \frac{d}{dz} (z - y^2) \right) + k \left(\frac{d}{dx} (x + z^3) - \frac{d}{dy} (z - y^2) \right)$$

$$= i(1 - 3z^2) + (1 - 2x)j + (1 + 2y)k = (1 - 3z^2, 1 - 2x, 1 + 2y)$$

$$\text{div}(F) = \frac{d}{dx} (z - y^2) + \frac{d}{dy} (x + z^3) + \frac{d}{dz} (y + x^2)$$

$$= 0 + 0 + 0 = 0$$

