Math 251 Shaun Goda Section 23 HW#8

January February March April May June July August September October November December

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

15.6:

1) a. $y = \frac{1}{2}x$ / u axis $y = x$ / v axis
b. parallelogram with points (0,0), (0,7), (10,2), (10,5)
c. segment between (2,3) and (10, 6)
d. triangle with points (0,1), (2,1), (2,2)

3) a. G is one-to-one on the domain $u \geq 0$ and $u \leq 0$
b. $[-1,1] \times [-1,1]$
c. x from 0 to 1 on the line $y = \sqrt{x}$
d. x from 0 to 1, y from 0 to 1, and $y = \sqrt{x}$

13) $x = 3u + 4v$, $y = u - 2v$

$\frac{\partial x}{\partial u} = 3 \quad \frac{\partial x}{\partial v} = 4 \quad \frac{\partial y}{\partial u} = 1 \quad \frac{\partial y}{\partial v} = -2$

$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} = (3)(-2) - (1)(4) = -6 - 4 = \boxed{-10}$

15) $x = r\sin(t) \quad y = r - \cos(t)$

$\frac{\partial x}{\partial r} = \sin(t) \quad \frac{\partial x}{\partial t} = r\cos(t) \quad \frac{\partial y}{\partial r} = 1 \quad \frac{\partial y}{\partial t} = \sin(t)$

$\frac{\partial(x,y)}{\partial(r,t)} = \begin{vmatrix} \sin(t) & r\cos(t) \\ 1 & \sin(t) \end{vmatrix} = \sin^2(t) - r\cos(t)$

$\sin^2(\pi) - (1)\cos(\pi) = \boxed{1}$

17) $x = r\cos\theta \quad y = r\sin\theta$

$\frac{\partial x}{\partial r} = \cos\theta \quad \frac{\partial x}{\partial \theta} = -r\sin\theta \quad \frac{\partial y}{\partial r} = \sin\theta \quad \frac{\partial y}{\partial \theta} = r\cos\theta$

$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = (\cos\theta)(r \overset{\cos}{\cancel{\sin}}\theta) - (\sin\theta)(-r \overset{\sin}{\cancel{\cos}}\theta)$

~~[illegible]~~ $= r\cos^2\theta + r\sin^2\theta$

~~[illegible]~~ $= r(\cos^2\theta + \sin^2\theta) = r = \boxed{4}$

19) $G(u,v) = (2v+4u,\ 3v+u)$

23) $x = 3u+v \quad y = u-2v$

$\frac{\partial x}{\partial u} = 3 \quad \frac{\partial x}{\partial v} = 1 \quad \frac{\partial y}{\partial u} = 1 \quad \frac{\partial y}{\partial v} = -2$

$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} = (3)(-2) - (1)(1) = -6 - 1 = -7$

Jacobian = 7

a. area of $R = [0,3] \times [0,5]$ is $3 \times 5 = 15$

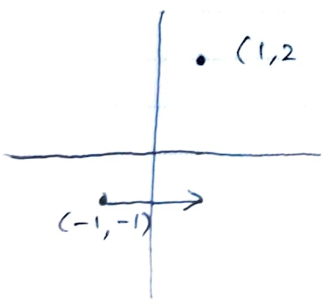
$15 \times 7 = 105$

b. area of $R = [2,5] \times [1,7]$ is $3 \times 6 = 18$

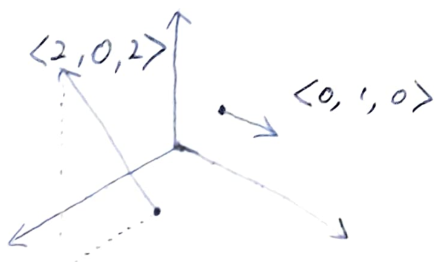
$18 \times 7 = 126$

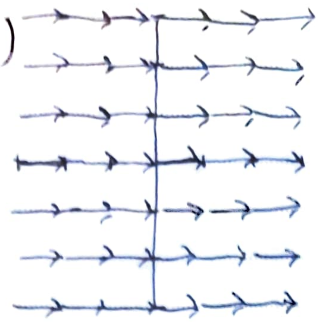
16.1:

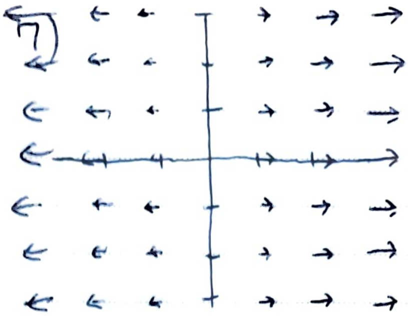
1) $F(1,2) = \langle 1, 2\rangle \quad F(-1,-1) = \langle 1, -1\rangle$

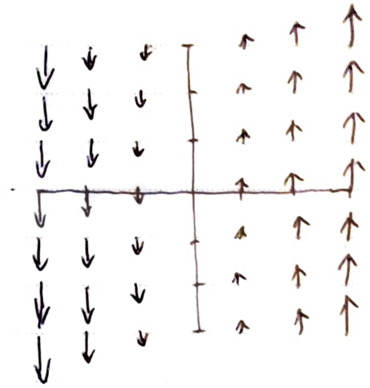


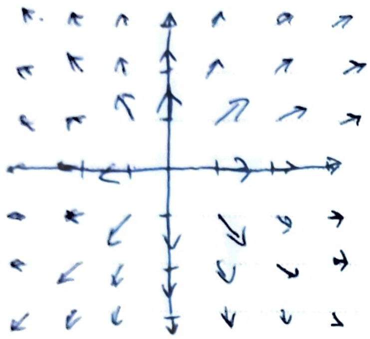
3) $F(0,1,1) = \langle 0,1,0\rangle \quad F(2,1,0) = \langle 2,0,2\rangle$



5) 

7) 

9) 

11) 

17) c

23) $\operatorname{div}(F) = \frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial y}(yz) + \frac{\partial}{\partial z}(y^2 - x^3)$

$= y + z$

$\operatorname{Curl}(F) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & y^2 - x^2 \end{vmatrix} = (2y - y)i + (0 + 2x)j + (0 - x)k$

$= \langle y, 2x, -x \rangle$

25) $\operatorname{div}(F) = \frac{\partial}{\partial x}(x - 2zx^2) + \frac{\partial}{\partial y}(z - xy) + \frac{\partial}{\partial z}(z^2x^2)$

$= (1 - 4zx) + (-x) + (2zx^2)$

$= 1 - x + 2zx(x - 2)$

$\operatorname{Curl}(F) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x - 2zx^2 & z - xy & z^2x^2 \end{vmatrix} = (0 - 1)i + (-2x^2 - 2xz^2)j + (-y - 0)k$

$= -i + (2x^2 - 2xz^2)j + (-y)k$

$= \langle -1, 2x^2 - 2xz^2, -y \rangle$

27) $\operatorname{div}(F) = \frac{\partial}{\partial x}(z - y^2) + \frac{\partial}{\partial y}(x + z^3) + \frac{\partial}{\partial z}(y + x^2) = 0$

$\operatorname{Curl}(F) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z - y^2 & x + z^3 & y + x^2 \end{vmatrix} = (1 - 3z^2)i + (1 - 2x)j + (1 + 2y)k$

$= \langle 1 - 3z^2, 1 - 2x, 1 + 2y \rangle$