HW due 11/8/20

15.6 1, 3, 13, 15, 17, 19, 23

16.1 1, 3, 5, 7, 9, 11, 17, 23, 25, 27

15.6

1a. $G(u,v) = (2u, u+v)$

$G(u,0) = (2u, u+0) = (2u, u)$

$(x,y) = (2u, u)$ $\boxed{y = \frac{x}{2}}$: u-axis

$G(0,v) = (0,v)$ $x = 0$ $\boxed{y\text{-axis}}$: v-axis

b. (0,0) (5,0) (5,7) (0,7)

$G(0,0) = (0,0)$ $G(5,0) = (10,5)$

$G(5,7) = (10,12)$ $G(0,7) = (0,7)$

parallelogram : (0,0) (10,5) (10,12) (0,7)

c. (1,2) (5,3)

$G(1,2) = (2,3)$ $G(5,3) = (10,8)$

d. (0,1) (1,0) (1,1)

$G(0,1) = (0,1)$ $G(1,0) = (2,1)$ $G(1,1) = (2,2)$

3a. $G(u,v) = (u^2, v)$

$G(u,0) = (u^2, 0)$ $G(0,v) = (0,v)$

u axis $\boxed{x\text{ Axis}}$ v-axis $\boxed{y\text{-axis}}$

b (-1,-1) (-1,1) (1,-1) (1,1)

$G(-1,-1) = (1,-1)$ $G(-1,1) = (1,1)$

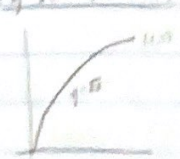
$G(1,-1) = (1,-1)$ $G(1,1) = (1,1)$

$[0,1] \times [-1,1]$

c $u = \sqrt{x}$ $v = y$

$\boxed{y = \sqrt{x} \quad 0 \le x \le 1}$

d



13. $Jac(G) = \frac{\partial(x,y)}{\partial(u,v)}$

$x = 3u + 4v$ $dx/du = 3$ $dx/dv = 4$

$y = u - 2v$ $dy/du = 1$ $dy/dv = -2$

$Jac(G) = \begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} = (-2)(3) - (1)(4)$

$= \boxed{-10}$

15. $x = r\sin t$ $dx/dr = \sin t$ $dx/dt = r\cos t$

$y = r - \cos t$ $dy/dr = 1$ $dy/dt = \sin t$

$Jac(G) = \begin{vmatrix} \sin t & r\cos t \\ 1 & \sin t \end{vmatrix} = \sin^2 t - r\cos t$

$Jac(G(1,\pi)) = 0 - (-1) = \boxed{1}$

17. $x = r\cos\theta$ $dx/dr = \cos\theta$ $dx/d\theta = -r\sin\theta$

$y = r\sin\theta$ $dy/dr = \sin\theta$ $dy/d\theta = r\cos\theta$

$Jac(G) = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta = r$

$Jac(G(4, \pi/6)) = \boxed{4}$

19. $G(u,v) = \langle Au + Cv, Bu + Dv \rangle$

$G(1,0) = (A,B) \to \langle 4,1 \rangle$

$G(0,1) = (C,D) \to \langle 2,3 \rangle$

$\boxed{G(u,v) = \langle 4u + 2v, u + 3v \rangle}$

23a $x = 3u + v$ $dx/du = 3$ $dx/dv = 1$

$y = u - 2v$ $dy/du = 1$ $dy/dv = -2$

$Jac(G) = \begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} = (3)(-2) - (1)(1) = \boxed{-7}$

$\int_0^5 \int_0^3 7 \, du\,dv = (7)(3)(5) = \boxed{105 \text{ sq. units}}$

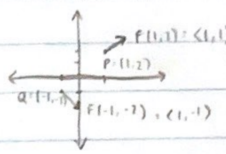
b. $\int_2^5 \int_1^7 (7) \, du\,dv = (7)(7-1)(5-2) = \boxed{126 \text{ sq. units}}$

16.1: 1, 3, 5, 7, 9, 11, 17, 23, 25, 27

16.1

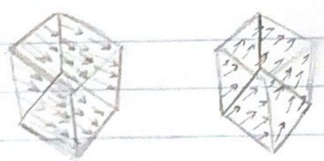
1. $F = \langle x^2, x \rangle$ $\quad P = (1, 2)$ $\quad Q = (-1, -1)$

$F(P) = \langle 1, 1 \rangle$ $\quad F(Q) = \langle 1, -1 \rangle$



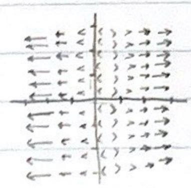
3. $F = \langle xy, z^2, x \rangle$ $\quad P = (0, 1, 1)$ $\quad Q = (2, 1, 0)$

$F(P) = \langle 0, 1, 0 \rangle$ $\quad F(Q) = \langle 2, 0, 2 \rangle$

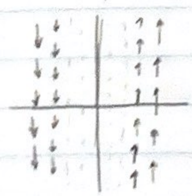


5.

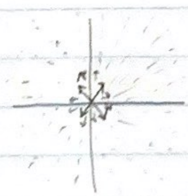
7.



9.



11.



17. C

23. $\operatorname{div} F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = \boxed{y + z}$

$\operatorname{curl} F = i\left(\frac{\partial}{\partial y}(y^2 - x^3) - \frac{\partial}{\partial z}(yz)\right) - j\left(\frac{\partial}{\partial x}(y^2 - x^3) - \frac{\partial}{\partial z}(xy)\right)$

$+ k\left(\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial y}(xy)\right)$

$= i(2y - y) - j(-3x^2) + k(-x)$

$= \boxed{yi + 3x^2 j - xk}$

25. $\operatorname{div} F = (1 - 4xz) + (-x) + 2zx^2 = \boxed{1 - 4xz - x + 2zx^2}$

$\operatorname{curl} F = i\left(\frac{d}{dy}(z^2x^2) - \frac{d}{dz}(z - xy)\right) - j\left(\frac{d}{dx}(z^2x^2) - \frac{d}{dz}(x - 2zx^2)\right)$

$+ k\left(\frac{d}{dx}(z - xy) - \frac{d}{dy}(x - 2zx^2)\right)$

$= i(0 - 1) - j(2z^2x + 2x^2) + k(-y - 0)$

$= \boxed{-i - (2z^2x + 2x^2)j - yk}$

27. $\operatorname{div} F = \boxed{0}$

$\operatorname{curl} F = i\left(\frac{d}{dy}(y + x^2) - \frac{d}{dz}(x + z^3)\right) - j\left(\frac{d}{dx}(y + x^2) - \frac{d}{dz}(z - y\right.$

$+ k\left(\frac{d}{dx}(x + z^3) - \frac{d}{dy}(z - y^2)\right)$

$= \boxed{i(1 - 3z^2) - j(2x - 1) + k(1 + 2y)}$