Calc HW Due

11/8

Rahul Paleja

Section 15.6 - #1, 3, 13, 15, 17, 19, 23:

Ask In Recitation ☆ (1) Determine the image under $G(u,v) = (2u, u+v)$ of the Following sets:

(a) The u- and v- axes

v must be 0 $\rightarrow \frac{2u}{2} = \frac{x}{2}$ $\boxed{u = \frac{1}{2}x}$

Image is line $y = \frac{1}{2}x$

v axis $\rightarrow 0 + v = y$ $\boxed{v = y}$

Image is y axis

(b) The rectangle $R = [0,5] \times [0,7]$ $G(u,v) = (2u, u+v)$

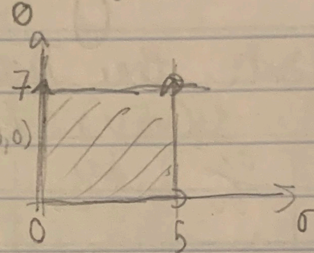
Parallelogram:

Vertices: $(0,0)$ $(2u, u+v) = (0,0)$

$(10,5)$

$(10,2)$

$(0,7)$



(c) Line Segment Joining $(1,2)$ & $(5,3)$ $G(u,v) = (2u, u+v)$

$(1,2) \rightarrow (2,3)$

$(5,3) \rightarrow (10,8)$ } segment joining $(2,3)$ & $(10,8)$

(d) Triangle With Vertices $(0,1), (1,0),$ & $(1,1)$

$G(0,1) = (0,1)$

$G(1,0) = (2,1)$

$G(1,1) = (2,2)$

Image of T is triangle with vertices $(0,1), (2,1),$ & $(2,2)$

(3) Let $G(u,v) = (u^2, v)$. Is G 1-to-1? If not, determine a domain on which G is one to one. Find the image under G.

- G is not 1 to 1 because it is possible to obtain the same outputs from different inputs where u can be positive or negative

(a) u and v axes $(u,0) = (u^2, 0) = x = u^2$; $y = 0$

$(0,v) = (0,v) = x = 0$; $y = v$

(b) Rectangle $R = [-1,1] \times [-1,1]$ $|u| \leq 1$, $|v| \leq 1$

$u^2 = x$ $v = y$ $|\sqrt{x}| \leq 1$

$u = \pm\sqrt{x}$ $v = y$ $|y| \leq 1$

4a

(c) line segment joining (0,0) and (1,1)

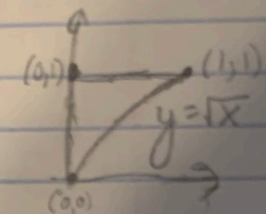
$G(u^2, v)$ $0 \le u^2 \le 1$ $0 \le v \le 1$

$u^2 = x$ $u = \sqrt{x}$ $v = y$ $\rightarrow$ $\boxed{y = \sqrt{x}}$

(d) Triangle with vertices (0,0), (0,1), and (1,1)

$G(u^2, v)$ <u>vertices of image:</u>

(0,0), (0,1) and (1,1)



$0 \le u^2 \le 1$ $u = \sqrt{x}$ $y = \sqrt{x}$

$0 \le v \le 1$ $v = y$

(13) Compute the Jacobian

$G(u,v) = (3u + 4v, u - 2v)$

$$\begin{vmatrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{vmatrix} \rightarrow \begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} \quad (3 \cdot -2) - 4 = \boxed{-10}$$

(15) $G(r,t) = (r\sin t, r - \cos t)$ $(r,t) = (1, \pi)$

$\partial x/\partial r = \sin(t)$ $\partial x/\partial t = r\cos t$ at $(1,\pi)$ $\partial x/\partial r = 0$ $\partial x/\partial t = -1$

$\partial y/\partial r = 1$ $\partial y/\partial t = \sin(t)$ $\partial y/\partial r = 1$ $\partial y/\partial t = 0$

$$\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = 0 - (-1) = \boxed{1}$$

(17) $G(r,\theta) = (r\cos\theta, r\sin\theta)$ $(r,\theta) = (4, \pi/6)$

$\partial x/\partial r = \cos\theta$ $\partial x/\partial\theta = -r\sin\theta$ at $(4, \pi/6) \rightarrow \begin{vmatrix} \sqrt{3}/2 & -2 \\ 1/2 & 2\sqrt{3} \end{vmatrix}$

$\partial y/\partial r = \sin\theta$ $\partial y/\partial\theta = r\cos\theta$

$3 + 1 = \boxed{4}$

Ask in Recitation (19) Map $[0,1] \times [0,1]$ to $\langle 2,3 \rangle$ and $\langle 4,1 \rangle$

$(0,1) = \langle 2,3 \rangle$

$\langle 0,1 \rangle = \langle B, D \rangle = \langle 2,3 \rangle$ $B = 2$ $D = 3$

$\langle A, C \rangle = \langle 4,1 \rangle$ $A = 4$ $C = 1$

$\boxed{(u,v) = \langle 4u + 2v, u + 3v \rangle}$

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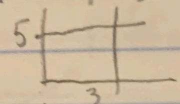
15.6 - #23:

23 Let $G(u,v) = (3u+v, u-2v)$. Use Jacobian to determine the area of $G(R)$ for:

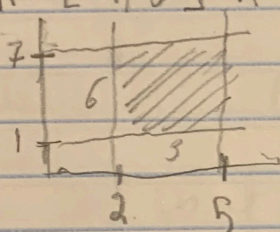
(a) $R = [0,3] \times [0,5]$



$$\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} = -6 - 1 = -7$$

Area $= |\text{Jacobian}| \cdot \text{Area}(R)$
$= 7 \cdot 15 = \boxed{105}$

(b) $R = [2,5] \times [1,7]$



Area $= 7 \cdot 18 = \boxed{126}$

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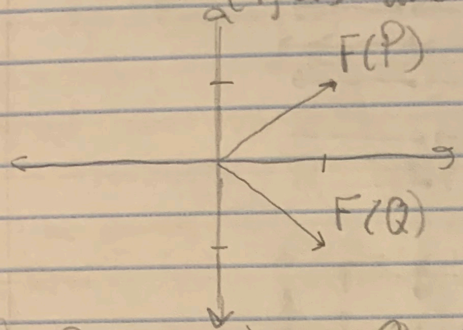
Section 16.1 - # 1, 3, 5, 7, 9, 11, 17, 23, 25, 27:

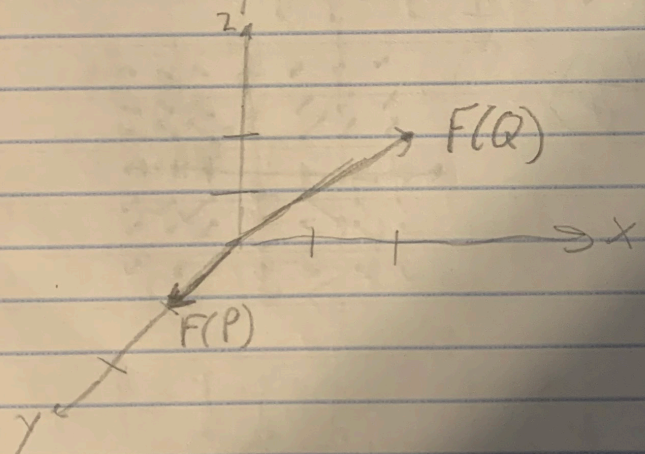
① Compute and sketch the vector assigned to the points $P=(1,2)$ and $Q=(-1,-1)$ by the vector field $F=\langle x^2, x\rangle$



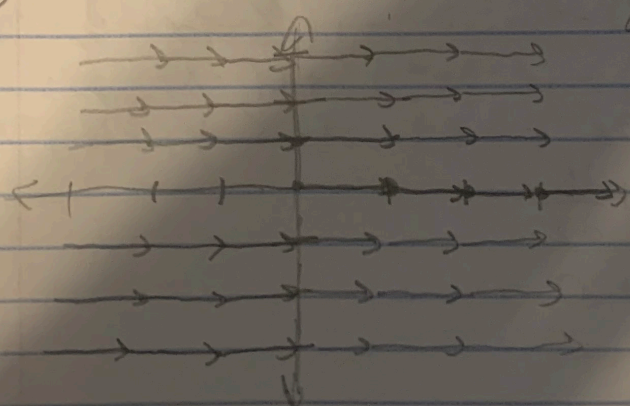
at point $P=(1,2)$
$F=\langle 1, 1\rangle$
at point $Q=(-1,-1)$
$F=\langle 1,-1\rangle$

③ $P=(0,1,1)$ $\quad Q=(2,1,0)$ $\quad F=\langle xy, z^2, x\rangle$

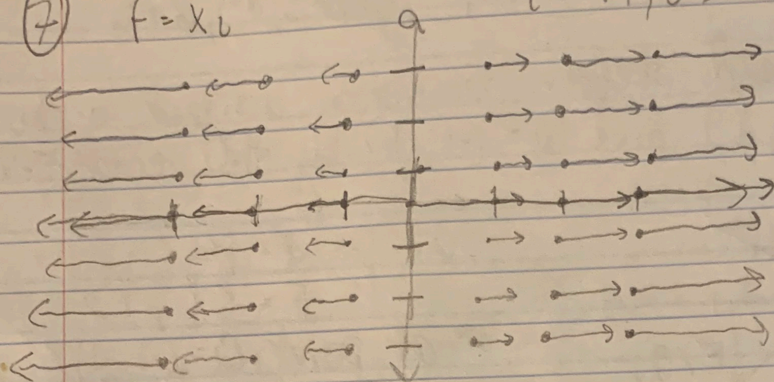
at point $P=(0,1,1) \rightarrow F=\langle 0,1,0\rangle$
at point $Q=(2,1,0) \rightarrow F=\langle 2,0,2\rangle$



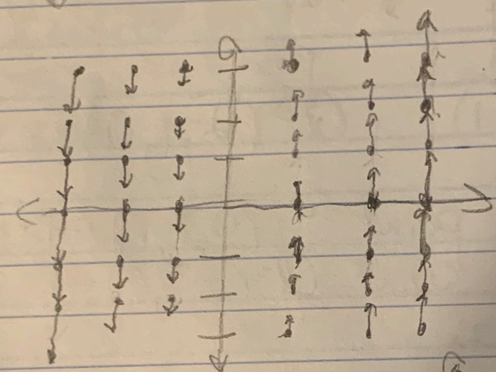
⑤ $F=\langle 1,0\rangle$ $\rightarrow$ A vector pointing one unit to right at every value $-3\leq x\leq 3$ and $-3\leq y\leq 3$



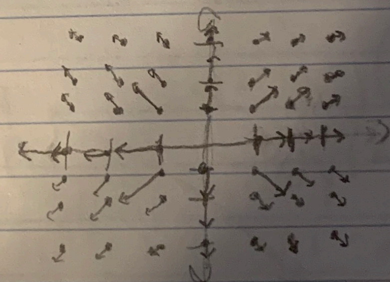
7. $F = x\vec{i}$ $\quad i = \langle 1, 0 \rangle$



9. $F = \langle 0, x \rangle$



11. $F = \left\langle \dfrac{x}{x^2+y^2}, \dfrac{y}{x^2+y^2} \right\rangle$



17. C

23. curl F $\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$ $\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy & yz & y^2-x^3 \end{vmatrix}$

$$i\left(\frac{\partial}{\partial y}(y^2-x^3) - \partial/\partial z(yz)\right) - j\left(\partial/\partial x(y^2-x^3) - \partial/\partial z(xy)\right) + k\left(\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial y}(xy)\right)$$

$$i(2y - y) - j(-3x^2) + k(-x)$$

$$= y\,i + 3x^2\,j - x\,k \rightarrow \boxed{\langle y, 3x^2, -x \rangle}$$

$$\operatorname{div} F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$= y + z + 0 = \boxed{y+z}$$

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16.1 - # 25, 27:

(25) $F = \langle x - 2zx^2, z - xy, z^2x^2 \rangle$

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x-2zx^2 & z-xy & z^2x^2 \end{vmatrix}$$

$\text{curl}(F) = i\left(\partial/\partial y(z^2x^2) - \partial/\partial z(z-xy)\right) - j\left(\partial/\partial x(z^2x^2) - \partial/\partial z(x-2zx^2)\right)$
$+ k\left(\partial/\partial x(z-xy) - \partial/\partial y(x-2zx^2)\right)$

$= i(-1) - j(2z^2x - 2x^2) + k(-y - 0)$

$= -i - j(2z^2x - 2x^2) - k(y)$

$\text{curl}(F) \langle -1, -2z^2x + 2x^2, -y \rangle$

$\text{div}(F) = 1 - 4zx - x + 2zx^2$

(27) $F = \langle z - y^2, x + z^3, y + x^2 \rangle$

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z-y^2 & x+z^3 & y+x^2 \end{vmatrix}$$

$i\left(\partial/\partial y(y+x^2) - \partial/\partial z(x+z^3)\right) - j\left(\partial/\partial x(y+x^2) - \partial/\partial z(z-y^2)\right)$
$+ k\left(\partial/\partial x(x+z^3) - \partial/\partial y(z-y^2)\right)$

$= i(1 - 3z^2) - j(2x - 1) + k(1 - 2y)$

$\text{curl}(F) = \langle 1 - 3z^2, -2x + 1, 1 - 2y \rangle$

$\text{div}(F) = 0 + 0 + 0 = 0$