Racnel Balji

November 8, 2020

15.6 => 1, 3, 13, 15, 17, 19, 23

16.1 => 1, 3, 5, 7, 9, 11, 17, 23, 25, 27

15.6

(1) $G(u,v) = (2u,\ u+v)$

a. u-axis => $y=\frac{1}{2}x$ v-axis => the y-axis

b. $R = [0,5] \times [0,7]$

Parallelogram → $(0,0)(10,5)(10,12)(0,7)$

c. The segment joining $(2,3)$ and $(10,8)$ (switch from x and to u and v)

d. The Δ with vertices $(0,1)(2,1)(2,2)$

(3) $G(u,v) = (u^2, v)$ G is NOT one to one

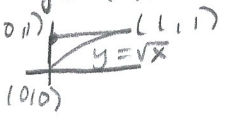
G domain $= \{(u,v): u \geq 0\}$ and $\{(u,v): u \leq 0\}$.

a. The posite x-axis and y-axis

b. $R = [-1,1] \times [-1,1]$ => $[0,1] \times [-1,1]$

c. $y = \sqrt{x}$

d.



(13) $G(u,v) = (3u+4v,\ u-2v)$

$$= \begin{vmatrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} = -6-4 = \boxed{-10}$$

$Jac(G) = -10$

(15) $G(r,t) = (r\sin t,\ r-\cos t)$ $(r,t) = (1,\pi)$

$$\begin{vmatrix} \frac{dx}{dr} & \frac{dx}{dt} \\ \frac{dy}{dr} & \frac{dy}{dt} \end{vmatrix} = \begin{vmatrix} \sin t & r\cos t \\ 1 & \sin t \end{vmatrix} = \sin^2 t - r\cos t$$

$= \sin^2\pi - (1)\cos\pi$

$= \boxed{1}$

(17) $G(r,\theta) = (r\cos\theta,\ r\sin\theta)$ $(r,\theta) = (4, \pi/6)$

$$\begin{vmatrix} \frac{dx}{dr} & \frac{dx}{d\theta} \\ \frac{dy}{dr} & \frac{dy}{d\theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta$$

$= 4\left(\frac{\sqrt{3}}{2}\right)^2 + 4\left(\frac{1}{4}\right)$

$= 3 + 1 = \boxed{4}$

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{3}{4} \cdot \frac{4}{1} = 3$

(19) $[0,1] \times [0,1]$

$\langle 2,3 \rangle$ and $\langle 4,1 \rangle$

$G(u,v) = (4u+2v,\ u+3v)$

(23) $G(u,v) = (3u+v,\ u-2v)$

$R = [0,3] \times [0,5]$

Jacobian => $\begin{vmatrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix}$

$= -6-1 = -7$

$= \int_0^3 \int_0^5 (-7)\, du\, dv$

$= -7v \Big|_0^5 = -35$

$\int_0^3 -35u \Big|_0^3 = -35(3) - (-35(0))$

$= \boxed{105 \text{ units}}$

$R = [2,5][1,7]$

$= \int_2^5 \int_1^7 (-7)\, dv\, du =$

$\int_1^7 (-7)\, dv = -7(7)+7 = -42$

$\int_2^5 (-42)\, du = -42(5) + 42(2)$

$= -126$

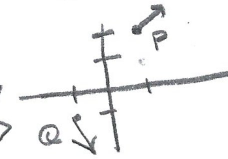
Area(G(R)) $= \boxed{126 \text{ units}}$

16.1

$F = \langle x^2, x \rangle$

(1) $P = (1,2) = \langle 1,1 \rangle$

$Q = (-1,-1) = \langle 1,-1 \rangle$

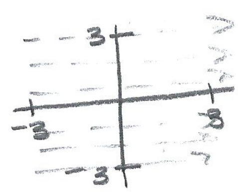


(3) $P = (0,1,1) = \langle 0,1,0 \rangle$

$Q = (2,1,0) = \langle 2,0,2 \rangle$ F(Q)

$F = \langle xy, z^2, x \rangle$

(5)

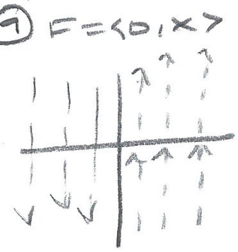


$F = \langle 1, 0 \rangle$

(7)

$F = xi$

(9) $F = \langle 0, x \rangle$



(11)

$F = \left\langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right\rangle$

232527 11/8/20

(17) $F = \langle 1, 1, 1 \rangle \Rightarrow C$

(23) div(F) and curl(F)

$\text{div}(F) = y + z$

$F = \langle xy, yz, y^2 - x^3 \rangle$

$$\text{curl}(F) = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ P & Q & R \end{vmatrix}$$

$$= \begin{vmatrix} i & j & k \\ \frac{d}{dx} & d/dy & d/dz \\ xy & yz & y^2 - x^3 \end{vmatrix} = (2y - y)i - (-3x^2 - 0)j + (0 - x)k$$

$= \langle y, 3x^2, -x \rangle = \text{curl}(F)$

(25) $F = \langle x - 2zx^2, z - xy, z^2x^2 \rangle$

$d/dx \quad d/dy \quad d/dz$

$$= \begin{vmatrix} x - 2zx^2 & x - xy & z^2x^2 \end{vmatrix} \Rightarrow (0 - 0)i - (2xz^2 + 2x^2)j + (1 - y)k$$

$= -i - (2xz^2 + 2x^2)j + (1 - y)k$

$F\text{curl} = \langle -1, 2x^2 - 2xz^2, -y \rangle$ $\quad \text{div}(F) = 1 - 4xz - x + 2x^2z$

(27) $F = \langle z - y^2, x + z^3, y + x^2 \rangle$

$\text{curl}F = (1 - 3z^2)i - (1 - 2x)j + (1 + 2y)k$

$=$

$\text{div}(F) = 0$

$\text{curl}(F) = \langle 1 - 3z^2, 1 - 2x, 1 + 2y \rangle$