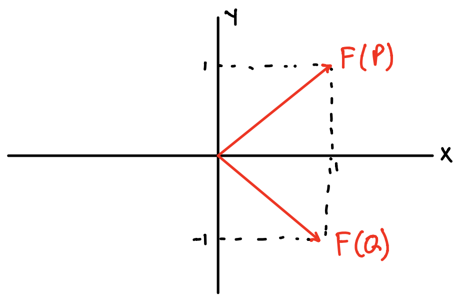
16.1 Homework

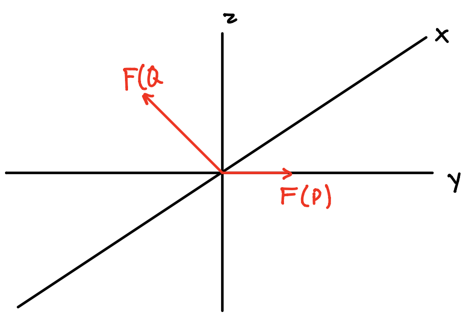
① $P = (1,2)$, $Q = (-1,-1)$, $F = \langle x^2, x \rangle$

$F(P) = \langle 1, 1 \rangle$, $F(Q) = \langle 1, -1 \rangle$

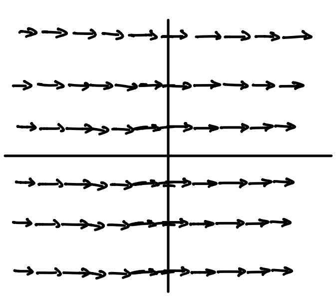


③ $P = (0,1,1)$, $Q = (2,1,0)$, $F = \langle xy, z^2, x \rangle$

$F(P) = \langle 0, 1, 0 \rangle$, $F(Q) = \langle 2, 0, 2 \rangle$

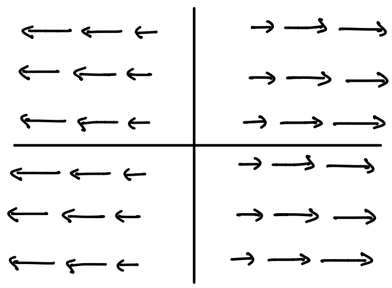


⑤

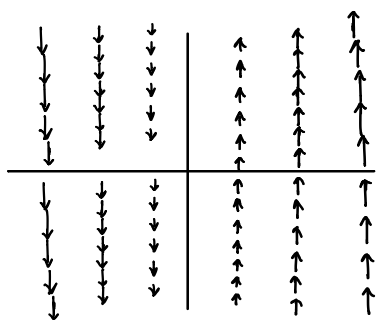


(7) $F = x\,i$

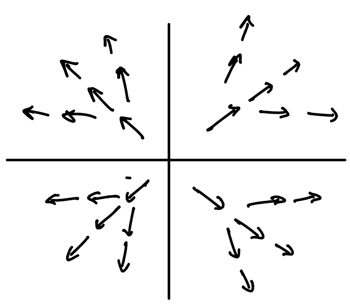
$F = \langle x, 0 \rangle$



(9) $F(x,y) = \langle 0, x \rangle$



(11) $F = \left\langle \dfrac{x}{x^2+y^2}, \dfrac{y}{x^2+y^2} \right\rangle$



(17) $F = \langle 1, 1, 1 \rangle$ C

(23) $F = \langle xy, yz, y^2 - x^3 \rangle$

$\langle P, Q, R \rangle$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & y^2 - x^3 \end{vmatrix} \Rightarrow \text{curl}(F) = \langle y, 3x^2, -x \rangle$$

$$\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \Rightarrow \text{div}(F) = y + z$$

(25) $F = \langle x - 2zx^2, z - xy, z^2x^2 \rangle$

$\langle P, Q, R \rangle$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x - 2zx^2 & z - xy & z^2x^2 \end{vmatrix} \Rightarrow \text{curl}(F) = \langle -1, -2x(x + z^2), -y \rangle$$

$$\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \Rightarrow \text{div}(F) = 2x^2z - x(4x + 1) + 1$$

(27) $F = \langle z - y^2, x + z^3, z^2x^2 \rangle$

$\langle P, Q, R \rangle$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z - y^2 & x + z^3 & z^2x^2 \end{vmatrix} \Rightarrow \text{curl}(F) = \langle -3z^2, 1 - 2xz^2, 2y + 1 \rangle$$

$$\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \quad \Rightarrow \operatorname{div}(F) = 2x^2 z$$

15.6 Homework

① $G(u,v) = (2u, u+v)$

a) u-axis: $y = \frac{1}{2}x$

v-axis: y-axis

b) Parallelogram $[(0,0), (10,5), (10,2), (0,7)]$

c) Line joining $(2,3)$ and $(10,8)$

d) Triangle $[(0,1), (2,1)]$

③ $G(u,v) = (u^2, v)$

a) $+x$-axis, $+y$-axis

b) $R = [0,1] \times [-1,1]$

c) $y = \sqrt{x}$ $[0 \le x \le 1]$

⑬ $G(u,v) = (3u+4v, u-2v)$

$x = 3u + 4v$, $y = u - 2v$

$$\begin{vmatrix} \frac{\partial(3u+4v)}{\partial u} & \frac{\partial(3u+4v)}{\partial v} \\ \frac{\partial(u-2v)}{\partial u} & \frac{\partial(u-2v)}{\partial v} \end{vmatrix} = -10$$

⑮ $G(r,t) = (r\sin t, r-\cos t)$, $(r,t) = (1,\pi)$

$x = r\sin t$, $y = r - \cos t$

$\mathrm{Jac}(G) = \sin^2 t - r\cos t = \sin^2\pi - \cos\pi = 1$

(17) $G(r,\theta) = (r\cos\theta, r\sin\theta)$, $(r,\theta) = (4, \frac{\pi}{6})$

$X = r\cos\theta$, $Y = r\sin\theta$

$Jac(G) = -r\sin^2\theta - r\cos^2\theta$

$Jac(G) = 4\sin^2\frac{\pi}{6} + 4\cos^2\frac{\pi}{6} = 4$

(19) G, $R = [0,1] \times [1,0]$, $\langle -2,5 \rangle$, $\langle 4,1 \rangle$

$G(u,v) = (au + bv, cu + dv)$

$G(0,1) = (2,3)$, $G(1,0) = (4,1)$

$(b,d) = (2,3)$ $\quad$ $(a,c) = (4,1)$

$G(u,v) = (4u + 2v, u + 3v)$

(23) $G(u,v) = (3u + v, u - 2v)$

$Jac(G) = -7$, $Area(G(R)) = 7\,Area(R)$

a) $R = [0,3] \times [0,5] \Rightarrow Area(R) = 15$

$Area(G(R)) = (7)(15) = 105$

b) $R = [2,5] \times [1,7] \Rightarrow Area(R) = 18$

$Area(G(R)) = (7)(18) = 126$