15.6: 1 3 13 15 17 19 23

Chap 15 HW Orion Kress-Sanfilippo

1) Det. img under $G(u, v) = (2u, u+v)$ of:

$$\left(\det \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = \boxed{2} \right)$$

a) u- and v- axes

Ans: u-axis corresponds to $y = 2x$; v-axis → y-axis

b) $R = [0, 5] \times [0, 7]$

Ans Shape w/ vertices: (0, 0), (0, 7), (10, 5), (10, 7)

c) Line segment {(1, 2)(5, 3)}

Ans → Line segment w/ points: {(2, 3), (10, 8)}

b) Triangle w/ vertices: {(0, 1), (1, 0), (1, 1)}

Ans → Triangle w/ vertices {(0, 1), (2, 1), (2, 2)}

15.6 Cont

3) $G(u, v) = (u^2, v)$ Only 1-to-1 on $u \geq 0$ OR $u \leq 0$

a) u- and v-axes

u-axis → pos x-axis

v-axis → y-axis

b) $R = [-1, 1] \times [-1, 1]$

Only $u \geq 0$

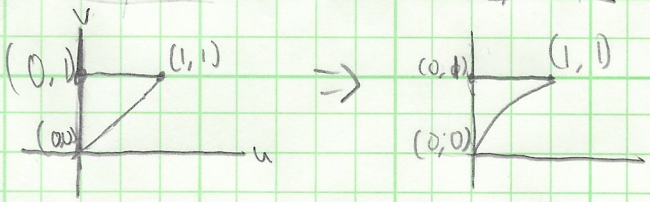
→ Rectangle formed by $[0, 1] \times [-1, 1]$

c) Line segment: $(0, 0)$ to $(1, 1)$

→ Same 2 pts, but along $y = \sqrt{x}$, not linear

d) Triangle: $\{(0,0), (0,1) \;\&\; (1,1)\}$

u-v plane → x-y plane



15.6 Cont

13) $G(u, v) = (3u + 4v,\ u - 2v)$

$$\text{Jac}(G) = \det\begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} = \boxed{-10}$$

15) $G(r, t) = (r \cdot \sin(t),\ r - \cos(t))$ $(r, t) = (1, \pi)$

$$\text{Jac}(G) = \det\begin{vmatrix} \sin(t) & r\cos(t) \\ 1 & \sin(t) \end{vmatrix} = \sin^2(t) - r\cos(t)$$

$@(r, t) = (1, \pi)$

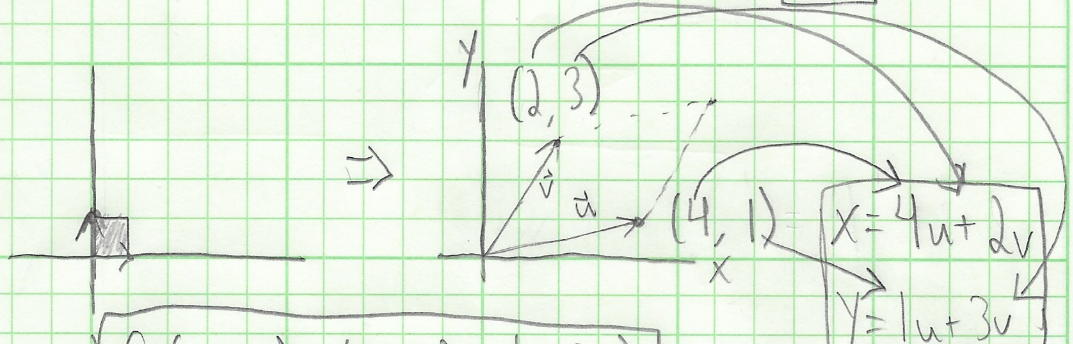
$$= 0 - 1(-1) = \boxed{1}$$

17) $G(r, \theta) = (r\cos\theta,\ r\sin\theta)$ $(r, \theta) = \left(4, \frac{\pi}{6}\right)$

$$\text{Jac}(G) = \det\begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\overset{1}{\cancel{(\cos^2\theta + \sin^2\theta)}}$$

$$= \boxed{4}$$

19)



$$\boxed{G(u, v) = (4u + 2v,\ 1u + 3v)}$$

15.6 Cont

23) $G(u,v) = (3u+v,\ u-2v)$

$$\mathrm{Jac}(G) = \det\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} = \boxed{+7}$$

a) $\text{Area}(R) = 3 \times 5 = 15$

$\text{Area}(G(R)) = (3 \times 5) \cdot 7 = 105$

b) $\text{Area}(R) = (3 \times 6) = 18$

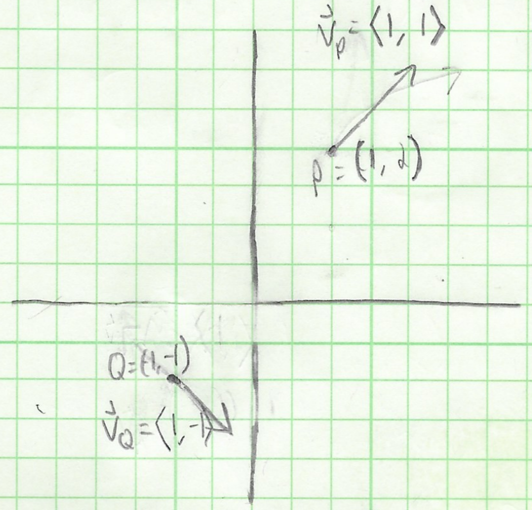
$\text{Area}(G(R)) = ((3 \times 6) \cdot 7) = 126$

16.1 - 1, 3, 5, 7, 9, 11, 17, 23, 25, 27

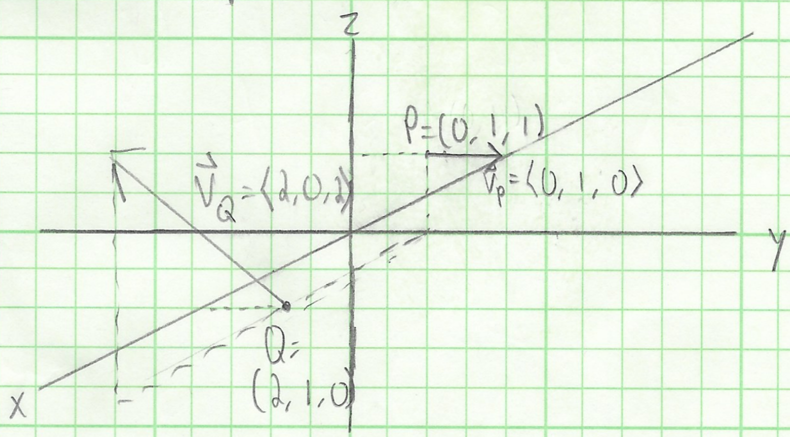
Chap 16 HW

16.1

1)



3)

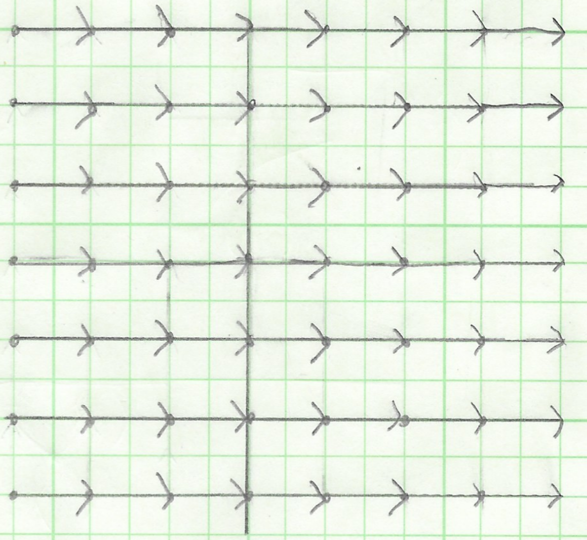


$F = \langle xy, z^2, x \rangle$ $\quad \vec{v}_P = \langle 0, 1, 0 \rangle$

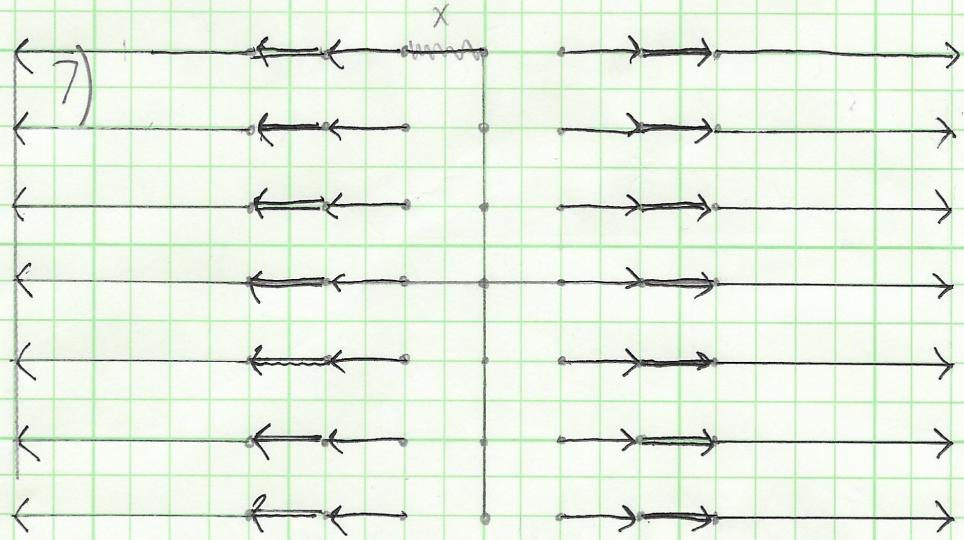
$\vec{v}_Q = \langle 2, 0, 2 \rangle$

16.1 Cont

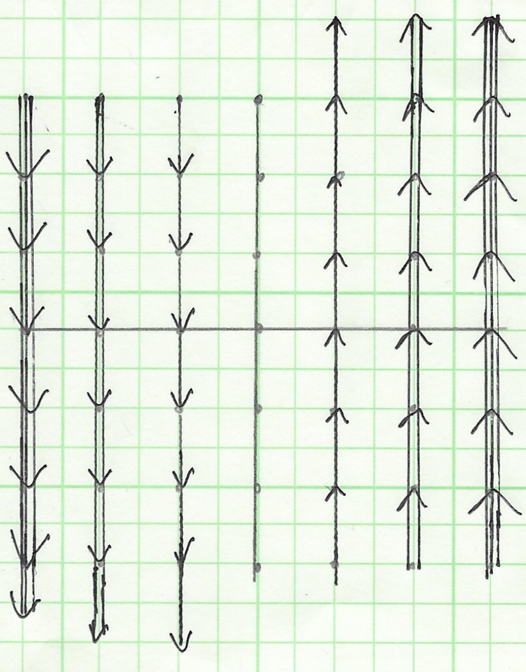
5)



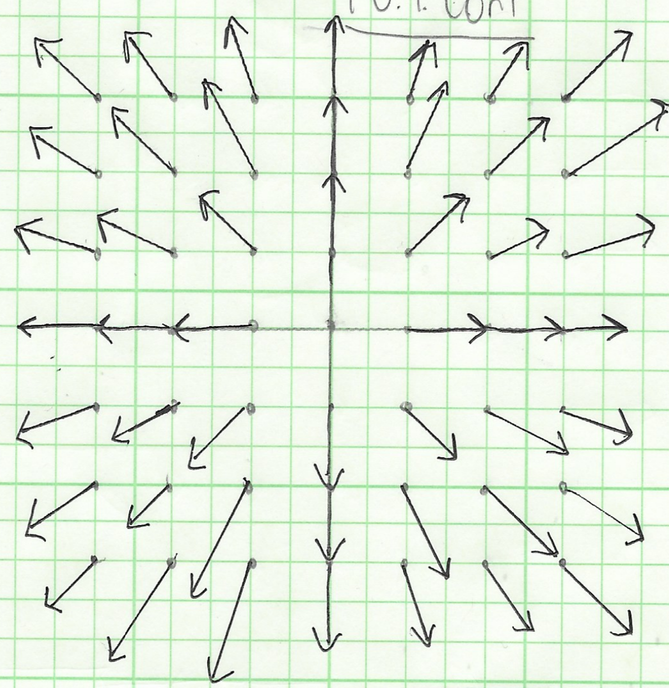
7)



9)



11)



~~Note: These are all (supposed to be) unit vectors. I got sloppy~~

17) $F = \langle 1, 1, 1 \rangle$ corresponds to plot (C)

For 23, 25, 27:

$$\boxed{\operatorname{div}(F) = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} \qquad \operatorname{curl}(F) = \det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}}$$

23) $F = \langle xy, yz, y^2 - x^3 \rangle$

$\operatorname{div}(F) = y + z + 0$

$\operatorname{curl}(F) = (2y - y)\hat{i} - (3x^2 - 0)\hat{j} + (0 - x)\hat{k}$

$= y\hat{i} - 3x^2\hat{j} - x\hat{k}$

16.1 Cont

25) $F = \langle x - 2zx^2,\ z - xy,\ z^2x^2 \rangle$

$$\operatorname{div}(F) = (1 - 4zx) + (-x) + (2zx^2)$$

$$\operatorname{curl}(F) = (0 - 1)\hat{i} - (2z^2x - (-2x^2))\hat{j} + (-y - 0)\hat{k}$$

$$= -\hat{i} - 4(z^2x + x^2)\hat{j} - y\hat{k}$$

27) $F = \langle z - y^2,\ x + z^3,\ y + x^2 \rangle$

$$\operatorname{div}(F) = 0 + 0 + 0 = 0$$

$$\operatorname{curl}(F) = (1 - 3z^2)\hat{i} - (2x - 1)\hat{j} + (1 - (-2y))\hat{k}$$

$$= (1 - 3z^2) + (1 - 2x)\hat{j} + (1 + 2y)\hat{k}$$