15.6.

1.

(a) For u-axies. $v=0$.

$G(u,v)=(2u,u)$

$y=\frac{1}{2}x$.

For v-axies. $u=0$.

$G(u,v)=(0,v)$

It's the y-axies.

(b) $(0,0)\rightarrow(0,0)$.

$(5,0)\rightarrow(10,5)$

$(0,7)\rightarrow(0,7)$

$(5,7)\rightarrow(10,12)$

So it's a parallelogram with these four points.

(c) The line is $y=\frac{1}{4}x+\frac{7}{4}$

$(1,2)\rightarrow(2,3)$

$(5,3)\rightarrow(10,8)$.

The line become $y=x-2$

(d) $(0,1)\rightarrow(0,1)$

$(1,0)\rightarrow(2,1)$

$(1,1)\rightarrow(2,2)$.

It become a triangle with these three points.

3. Consider there's a $u^2$.

It's only one-to-one when $u\geq0$ or $u\leq0$.

(a) For u-axies, $v=0$.

$G(u,v)=(u^2,0)$

It's the ~~posive~~ positive x-axies.

For v-axies, $u=0$.

$G(u,v)=(0,v)$

It's still the y-axies.

(b). $(-1,-1)\rightarrow(1,-1)$

$(1,-1)\rightarrow(1,-1)$

$(-1,1)\rightarrow(1,1)$

$(\cancel{0}1,1)\rightarrow(1,1)$.

It's $[0,1]\times[-1,1]$

(c) The line is $y=x$.

It become $y^2=x$, $y=\sqrt{x}\ (0\leq x\leq1)$.

(d) $(0,0)\rightarrow(0,0)$

$(0,1)\rightarrow(0,1)$

$(1,1)\rightarrow(1,1)$

~~It's~~ The line between $(0,0)$ and $(1,1)$ become $y=\sqrt{x}$. others no change.

13 Jacobian: $\cancel{4\times1+3\times2=10}$.

$3\times(-2)-4\times1=-10$.

15. $\sin^2 t-r\cos t$.

when $\langle r,t\rangle=\langle1,\pi\rangle$

Jacobian: $\sin^2\pi-\cos\pi=1$.

17. $r\cos^2\theta + r\sin^2\theta = r$.

when $\langle r,\theta\rangle = \langle 4, \frac{\pi}{6})$

Jacobian $= 4$

19. It's the form of

$(au+bv,\ cu+dv)$.

Because $\langle 2,3\rangle$.

$a=2$ and $c=3$

$\because \langle 4,1\rangle$

$b=4$ and $d=1$.

$G(u,v) = (2u+4v,\ 3u+v)$.

Jacobian: $3\times(-2) - 1\times 1 = -7$

23.(a) $\cancel{(0,0) \to (0,0)}$

$\cancel{(3,0) \to (9,3)}$

$\cancel{(0,5) \to (5,-10)}$

$\cancel{(3,5) \to (14,-7)}$

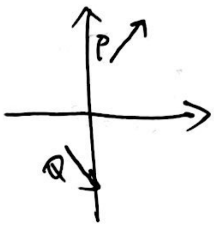
$(3-0)\times(5-0)\times|-7| = 105$.

(b) $(5-2)\times(7-1)\times|-7| = 126$

16.1.

1. $F(P) = \langle 1,1\rangle$

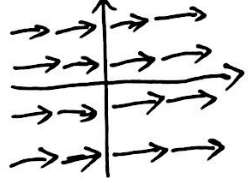
$F(Q) = \langle 1,-1\rangle$



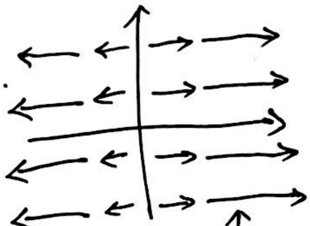

3. $F(P) = \langle 0,1,0\rangle$

$F(Q) = \langle 2,0,2\rangle$.

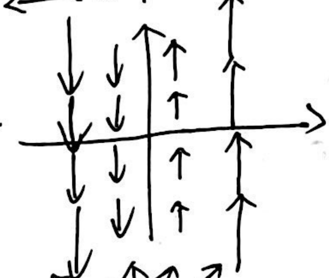
5.



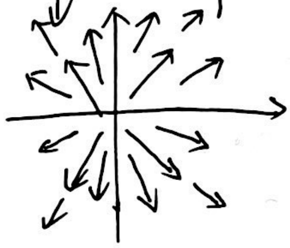
7.



9.



11.



17. It's C. All in same direction

23. $\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & y^2-x^3 \end{vmatrix}$

$\mathrm{Curl}(F) = i\left(\frac{\partial}{\partial y} y^2-x^3 - \frac{\partial}{\partial z} yz\right)$

$- j\left(\frac{\partial}{\partial x} y^2-x^3 - \frac{\partial}{\partial z} xy\right)$

$+ k\left(\frac{\partial}{\partial x} yz - \frac{\partial}{\partial y} xy\right)$

$= yi + 3x^2 j - xk$

$\mathrm{div}(F) = \frac{\partial}{\partial x} xy + \frac{\partial}{\partial y} yz + \frac{\partial}{\partial z} y^2-x^3$

$= y + z$

25. $\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2-2x^2z & z-xy & z^2x^2 \end{vmatrix}$

$$\begin{aligned}\text{Curl}(F) &= i\left(\frac{\partial}{\partial y}x^2z^2 - \frac{\partial}{\partial z}z-xy\right) \\ &\quad - j\left(\frac{\partial}{\partial x}x^2z^2 - \frac{\partial}{\partial z}x^2-2x^2z\right) \\ &\quad + k\left(\frac{\partial}{\partial x}z-xy - \frac{\partial}{\partial y}x^2-2x^2z\right) \\ &= -i+(2x^2-2x)j-yk\end{aligned}$$

$$\begin{aligned}\text{div}(F) &= \frac{\partial}{\partial x}x^2-2x^2z + \frac{\partial}{\partial y}z-xy + \frac{\partial}{\partial z}x^2z^2 \\ &= \overset{1}{\cancel{2x}}-4xz-x+2x^2z\end{aligned}$$

27. $\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z-y^2 & x+z^3 & y+x^2 \end{vmatrix}$

$$\begin{aligned}\text{Curl}(F) &= i\left(\frac{\partial}{\partial y}y+x^2 - \frac{\partial}{\partial z}x+z^3\right) \\ &\quad - j\left(\frac{\partial}{\partial x}y+x^2 - \frac{\partial}{\partial z}z-y^2\right) \\ &\quad + k\left(\frac{\partial}{\partial x}x+z^3 - \frac{\partial}{\partial y}z-y^2\right) \\ &= (1-3z^2)i+(1-2x)j+(2y+1)k\end{aligned}$$

$$\text{div}(F) = \frac{\partial}{\partial x}z-y^2 + \frac{\partial}{\partial y}x+z^3 + \frac{\partial}{\partial z}y+x^2 = 0.$$