

15.6

$$1) G(u, v) = (2u, u+v)$$

$$x = 2u$$

$$y = u+v$$

$$u = \frac{x}{2}$$

$$(b) R = [0, 5] \times [0, 7]$$

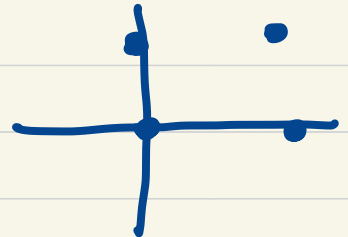
$$0 \leq x \leq 5$$

$$0 \leq y \leq 7$$

$$(0, 0), (0, 7), (5, 0), (5, 7)$$

$$= (0, 0), (0, 7), (10, 5)$$

$$(10, 12)$$



$$(E) \quad (0,1) \quad (1,0) \quad (1,1) \quad 2u, u+v$$
$$= (0,1) \quad (2,1) \quad (2,2)$$

$$3) \quad G(u,v) = (u^2, v)$$

G is not one-one

For one to one, $u \geq 0$ & $u \leq 0$

(b) Rectangle $[-1,1] \times [-1,1]$

$$(-1,1) \quad (-1,1) \quad (1,-1) \quad (1,1)$$

$$(1,1) \quad (1,1) \quad (1,-1) \quad (1,1)$$

$$(c) \quad (0,0) \quad (1,1)$$

$$(d) \quad (0,0) \quad (0,1) \quad (1,1)$$

$$\rightarrow (0,0) \quad (0,1) \quad (1,1)$$

same

$$13) \quad f(3u+4v, u-2v)$$

$$x = 3u + 4v \quad y = u - 2v$$

$$\begin{vmatrix} dx/du & dx/dv \\ dy/du & dy/dv \end{vmatrix}$$

$$\begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} = -6 - 4 \\ = -10$$

$$15) \quad x = r \sin t$$

$$y = r - \cos t$$

$$\begin{vmatrix} \frac{dx}{dr} & \frac{dx}{dt} \\ \frac{dy}{dr} & \frac{dy}{dt} \end{vmatrix}$$

$$= \begin{vmatrix} \sin t & r \cos t \\ 1 & r + \sin t \end{vmatrix}$$

$$\begin{aligned} r &= 1 \\ t &= \pi \end{aligned}$$

$$= \begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= 1$$

$$17) \quad x = r \cos \theta$$

$$y = r \sin \theta$$

$$\frac{30}{\cancel{180}} \\ 61$$

$$\begin{vmatrix} \frac{dx}{dr} & \frac{dx}{d\theta} \\ \frac{dy}{dr} & \frac{dy}{d\theta} \end{vmatrix}$$

$$\begin{vmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{vmatrix}$$

$$r = 4$$

$$\theta = \pi/6$$

$$r \cos^2 \theta + r \sin^2 \theta$$

$$r = 4$$

$$19) [0,1] \times [0,1]$$

$$[0,0] \quad [0,1] \quad [1,0] \quad [1,1]$$

$$\langle 2, 3 \rangle \quad \langle 4, 1 \rangle$$

$$4u + 2v$$

$$u + 3v$$

$$23) G(u, v) = (3u + v, u - 2v)$$

$$R = [0, 3] \times [0, 5]$$

$$= (0, 0) \quad (0, 5) \quad (3, 0) \quad (3, 5)$$

$$(0, 0), (5, -10), (6, 3), (14, -7)$$

$$\text{Area} = 105$$