homework 9

15.6

1.) $G(u,v) = (2u, u+v)$

a.) ~~$y = x = 2u$~~ $y = \frac{1}{2}x$, ~~$y=0$~~ ~~$x=0$~~

U: $y = \frac{1}{2}x$, r: $x = 0$ (y-axis)

b.) $[0,5] \times [0,7]$

now $5 = \frac{1}{2}x$ $\left\{ \begin{array}{l} 0 = \frac{1}{2}x \\ x = 0 \end{array} \right.$ $\left\{ \begin{array}{l} 7 = 0 + y \\ y = 7 \end{array} \right.$

$x = 10$

$(10,5)$ $(0,0)$ $(0,7)$ $(10,2)$

c.) ~~boundary points~~

$(1,2)$ $(5,3)$

$2(1) = 2$ $\{ 5(2), 5+3$

$(2,3)$ $(10,8)$

d.) $(0,1)$ $(1,0)$ $(1,1)$

$(0,1)$ $\{ (2,1)$ $\{ 2(1), 2(1)$

$(0,1)$ $(2,1)$ $(2,2)$

3.) $G(u,v) = (u^2, v)$

→ not 1:1, D: $u \geq 0$, $u \leq 0$

a) x-axis, y-axis

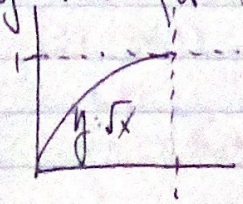
b.) $R = [-1,1] \times [-1,1]$

$[-1,1] \times [0,1]$

c.) → function on bot

$y = \sqrt{x}$ for $x \leq 1$ and $x \geq 0$

d.)



13.) $G(u,v) = (3u+4v,\ u-2v)$

$$\begin{vmatrix} \frac{\partial(3u+4v)}{\partial u} & \frac{\partial(3u+4v)}{\partial v} \\ \frac{\partial(u-2v)}{\partial u} & \frac{\partial(u-2v)}{\partial v} \end{vmatrix} = -10$$

15.) $G(r,t) = (r\sin t,\ r-\cos t)$

$(r,t) = (1, \pi)$

$0, 2$

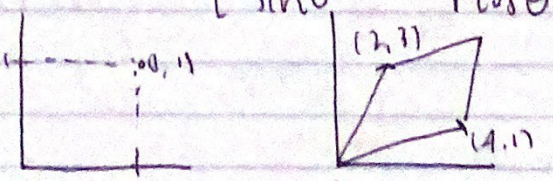
$$\begin{bmatrix} \frac{\partial(r\sin t)}{\partial r} & \frac{\partial(r\sin t)}{\partial t} \\ \frac{\partial(r-\cos t)}{\partial r} & \frac{\partial(r-\cos t)}{\partial t} \end{bmatrix} = 1$$

17.) $G(r,\theta) = (r\cos\theta,\ r\sin\theta)$

$(4, \frac{\pi}{6})$

$$\begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix} = r \to 4$$

19.)



$\varphi(u,v) = (Au+Bv,\ Cu+Dv)$

$\varphi(0,1) = (2,3)$

$B = 1,\ D = 3$

$\varphi(1,0) = (4,1)$

$A = 4,\ C = 1$

$\varphi(u,v) = (4u+2v,\ u+3v)$

23.) $3u+v$, $u-2v$

$$\begin{bmatrix} \frac{\partial(3u+v)}{\partial u} & \frac{\partial(3u+v)}{\partial v} \\ \frac{\partial(u-2v)}{\partial u} & \frac{\partial(u-2v)}{\partial v} \end{bmatrix} = -7$$

$0 < 3u+v$ $\quad 0 < u-2v$

$-3 > 3u+v$ $\quad 0 > u-2v$

$5 > u+2v$

$x = 3u+v$

$y = u-2v$

$v > -3u$

$v < -3u+3$

3-0, 5-0

$3(5) + 15(7) = 105$

$3(6) + 18(7) = 126$

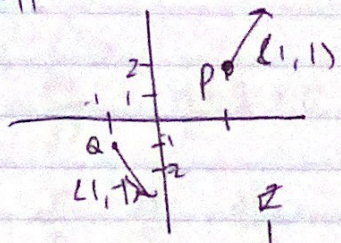
10.1

1.) $P = (1,2)$ $Q = (-1,-1)$

$F = \langle x^2, x \rangle$

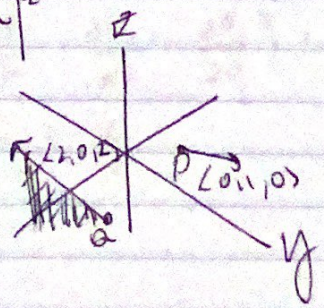
$F = \langle 1, 1 \rangle$

$F = \langle 1, -1 \rangle$

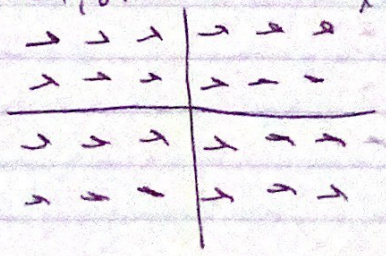


3.) $P = \langle 0, 1, 0 \rangle$

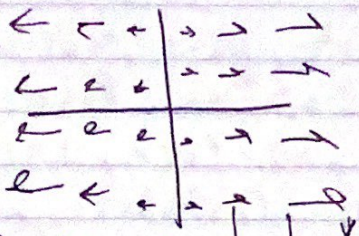
$Q = \langle 2, 0, 2 \rangle$



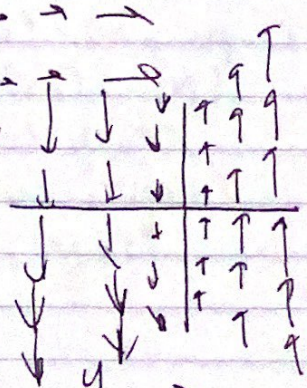
5.) $F = \langle 1, 0 \rangle$



7.) $F = xi$

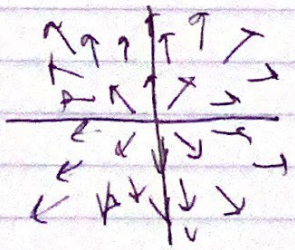


9.) $F(x,y) = \langle 0, x \rangle$



11.) $F = \left\langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right\rangle$

going away from origin & slowing rate



17.) $F = \langle 1, 1, 1 \rangle$

(C)

23.) $F = \langle xy, yz, y^2 - x^3 \rangle$

$$\text{div} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = y + z + 0$$

$$\text{curl} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) =$$

$$2y - y, \quad 0 - 3x^2, \quad 0 - y$$

$$(y, -3x^2, -y)$$

$$\text{curl} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)$$

25.) $F = \langle x - 2zx^3, z - xy, z^2x^2 \rangle$

div $= (1 - 6zx^2, -x, 2z)$

curl $= (0 - 1, -2x^3 - 2x, -y - 0)$

$= (-1, -2x(x^2 - 1), -y)$

27.) $F = \langle z - y^2, x + z^3, y + x^2 \rangle$

$= (1 - 3z^2, 1 + 0, y + 1)$

$= (1 - 3z^2, 1, y + 1)$