15.6 : 1, 3, 13, 15, 17, 19, 23

1) $G(u,v) = (2u, u+v)$

a) u- and v- axes

$x = 2u$ $\quad$ $x = 2\cancel{u}$

$y = u+v$ $\quad$ $-2y = -2\cancel{u} - 2v$

$x - 2y = -2v$

$x = 2y$

v → y-axis $\quad$ u → $y = \frac{1}{2}x$

b) rectangle $R = [0,5] \times [0,7]$

$x = 2u$ $\quad$ $y = u+v$

$x = 2(0) = 0$ $\quad$ $y = 0+0 = 0$

$x = 2(5) = 10$ $\quad$ $y = 0+5 = 5$

$x = 2(0) = 0$ $\quad$ $y = 0+7 = 7$

(0,0) (10,5) (10,2) (0,7)

c) line segment (1,2) and (5,3)

(2, 3)

(10, 8)

d) Δ vertices (0,1) (1,0) and (1,1)

(0,1)

(2,1)

(2,2)

3) $G(u,v) = (u^2, v)$ → not one-to-one → $\{(u,v) \mid u \geq 0\}$

a) u- and v-axes

u → x-axis

v → y-axis

b) rectangle $R = [-1,1] \times [-1,1]$

$[0,1] \times [-1,1]$

c) line segment of (0,0) and (1,1)

(0,0) (1,1)

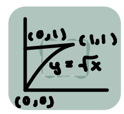
$y = \sqrt{x}$ for $0 \leq x \leq 1$

d) Δ vertices (0,0) (0,1) and (1,1)

(0,0)

(0,1)

(1,1)



13) $G(u,v) = (3u+4v, u-2v)$

$x_u = 3$ $\quad$ $x_v = 4$

$y_u = 1$ $\quad$ $y_v = -2$

$$\begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} = 3(-2) - 4(1) = -6 - 4 = -10$$

17) $G(r,\theta) = (r\cos\theta, r\sin\theta)$, $(r,\theta) = (4, \frac{\pi}{6})$

$x_r = \cos\theta = \frac{\sqrt{3}}{2}$ $\quad$ $x_\theta = -r\sin\theta = -2$

$y_r = \sin\theta = \frac{1}{2}$ $\quad$ $y_\theta = r\cos\theta = 2\sqrt{3}$

$$\begin{vmatrix} \frac{\sqrt{3}}{2} & -2 \\ \frac{1}{2} & 2\sqrt{3} \end{vmatrix}$$

$= \frac{\sqrt{3}}{2}(2\sqrt{3}) - (-2)(\frac{1}{2})$

$= 3 + 1 = 4$

19) Find linear mapping G of $[0,1] \times [0,1]$ to the parallelogram in xy-plane with vectors $\langle 2,3 \rangle$ and $\langle 4,1 \rangle$

$$\begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} = 0,1 \ \& \ 1,0$$

x

$2 = a(0) + b(1)$

$4 = a(1) + b(0)$

↓

$2 = b$

$4 = a$

$4u + 2v$

y

$3 = c(0) + d(1)$

$1 = c(1) + d(0)$

↓

$3 = d$

$1 = c$

$u + 3v$

$G(u,v) = (4u+2v, u+3v)$

23) $G(u,v) = (3u+v, u-2v)$

a) $R = [0,3] \times [0,5]$

$$\begin{vmatrix} 0 & 3 \\ 0 & 5 \end{vmatrix} = 0,5 \ \& \ 3,0$$

$G(0,5) = (5, -10)$

$G(3,0) = (9, 3)$

$$\begin{vmatrix} 5 & -10 \\ 9 & 3 \end{vmatrix}$$

$= 5(3) - (-10)(9) = 105$

b) $R = [2,5] \times [1,7]$

$$\begin{vmatrix} 2 & 5 \\ 1 & 7 \end{vmatrix} = 2,7 \ \& \ 5,1$$

$G(2,7) = (13, -12)$

$G(5,1) = (16, 3)$

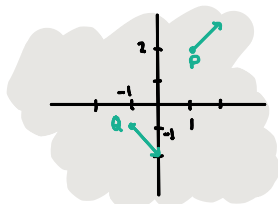
$$\begin{vmatrix} 13 & -12 \\ 16 & 3 \end{vmatrix}$$

$= 13(3) - (-12)(16) = 126$

16.1 : 1, 3, 5, 7, 9, 11, 17, 23, 25, 27

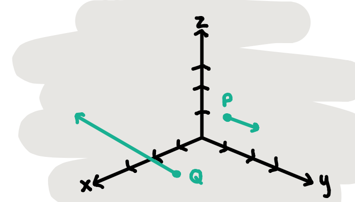
1) $P=(1,2)$ $Q=(-1,-1)$; vector field $F=\langle x^2, x\rangle$

$F(1,2)=\langle 1,1\rangle$ $F(-1,-1)=\langle 1,-1\rangle$



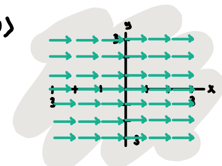
3) $P=(0,1,1)$ $Q=(2,1,0)$; $F=\langle xy, z^2, x\rangle$

$F(0,1,1)=\langle 0,1,0\rangle$ $F(2,1,0)=\langle 2,0,2\rangle$



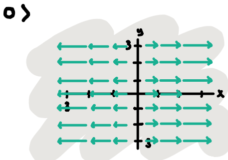
5) Rectangle : $-3\le x\le 3, -3\le y\le 3$

$F=\langle 1,0\rangle$



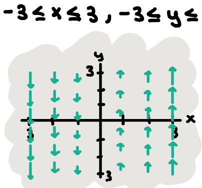
7) Rectangle : $-3\le x\le 3, -3\le y\le 3$

$F=xi \rightarrow \langle x,0\rangle$



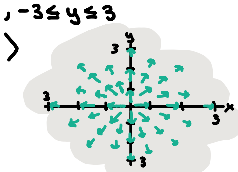
9) Rectangle : $-3\le x\le 3, -3\le y\le 3$

$F=\langle 0,x\rangle$



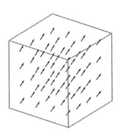
11) Rectangle : $-3\le x\le 3, -3\le y\le 3$

$F=\left\langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}\right\rangle$



17) match $F=\langle 1,1,1\rangle$ to image

(C)



(C)

23) $F=\langle xy, yz, y^2-x^3\rangle$

$\text{div}(F)=\left\langle \frac{\partial}{\partial x}(xy), \frac{\partial}{\partial y}(yz), \frac{\partial}{\partial z}(y^2-x^3)\right\rangle$

$=\langle y, z, 0\rangle$

$\text{curl}(F)=\nabla\times F$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & y^3-x^3 \end{vmatrix} = i\left(\frac{\partial}{\partial y}(y^3-x^3)-\frac{\partial}{\partial z}(yz)\right) - j\left(\frac{\partial}{\partial x}(y^3-x^3)-\frac{\partial}{\partial z}(xy)\right) + k\left(\frac{\partial}{\partial x}(yz)-\frac{\partial}{\partial y}(xy)\right)$$

$=i(3y^2-y)-j(-3x^2-0)+k(0-x)$

$=\langle 3y^2-y, 3x^2, -x\rangle$

25) $F=\langle x-2zx^2, z-xy, z^2x^2\rangle$

$\text{div}(F)=\left\langle \frac{\partial}{\partial x}(x-2zx^2), \frac{\partial}{\partial y}(z-xy), \frac{\partial}{\partial z}(z^2x^2)\right\rangle$

$=\langle 1-4zx, -x, 2zx^2\rangle$

$\text{curl}(F)=\nabla\times F$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x-2zx^2 & z-xy & z^2x^2 \end{vmatrix} = i\left(\frac{\partial}{\partial y}(z^2x^2)-\frac{\partial}{\partial z}(z-xy)\right) - j\left(\frac{\partial}{\partial x}(z^2x^2)-\frac{\partial}{\partial z}(x-2zx^2)\right) + k\left(\frac{\partial}{\partial x}(z-xy)-\frac{\partial}{\partial y}(x-2zx^2)\right)$$

$=i(0-1)-j(2z^2x-2x^2)+k(-y-0)$

$=\langle -1, -2z^2x+2x^2, -y\rangle$

27) $F=\langle e^y, \sin x, \cos x\rangle$

$\text{div}(F)=\left\langle \frac{\partial}{\partial x}(e^y), \frac{\partial}{\partial y}(\sin x), \frac{\partial}{\partial z}(\cos x)\right\rangle$

$=\langle 0,0,0\rangle = 0$

$\text{curl}=\nabla\times F$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^y & \sin x & \cos x \end{vmatrix} = i\left(\frac{\partial}{\partial y}(\cos x)-\frac{\partial}{\partial z}(\sin x)\right) - j\left(\frac{\partial}{\partial x}(\cos x)-\frac{\partial}{\partial z}(e^y)\right) + k\left(\frac{\partial}{\partial x}(\sin x)-\frac{\partial}{\partial y}(e^y)\right)$$

$=i(0-0)-j(-\sin x-0)+k(\cos x-e^y)$

$\langle 0, \sin x, \cos x - e^y\rangle$