11/6/20 15.6 Change of Variables.

15.6 #1, 3, 13, 15, 17, 19, 23

1) $G(u,v) = (2u, u+v)$

a) u-axis : $y = \frac{1}{2}x$, V-axis : -y axis.

b). Quadrilateral with vertices (0,0, (0,5)(10,2)(0,7)

c) (0,0) to (1,1) → (0,0) to (2,2)

d) Point (0,1), (2,1), (2,2))

3) $G(u,v) = (u^2, v)$ → not one to one. (u,v) and (-u, v) are the same. Domain: $u \geq 0$

a) U-xais image: $x = u^2$, $y = 0$

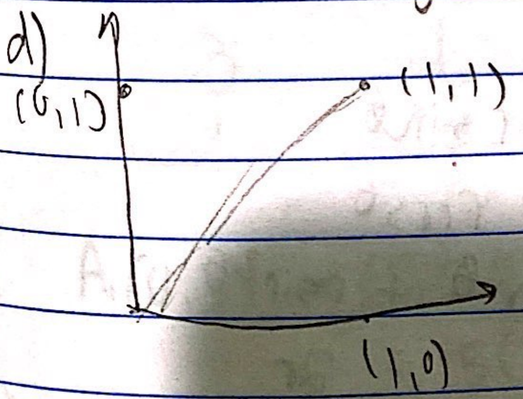
V-axis image: $y = v$

b) V-is outside our domain. If $x = u^2$, $\sqrt{x} = \pm u$.

$\sqrt{x} \leq 1$, $y \leq 1$

c) The curve $y = \sqrt{x}$ from [0, 1)

d)



$r\sin t$ $\quad$ $r\cos t$

13) $G(u,v) = (3u+4v,\ u - 2v)$

$$\begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} \quad -6-4 = -10$$

15) $G(r,t) = (r\sin t,\ r - r\cos t)$

$$= \begin{vmatrix} \sin t & r\cos t \\ 1-\cos t & r\sin t \end{vmatrix}$$

$(r\cos t)(1-\cos t)$

$r\cos t - r\cos^2 t$

$r\sin^2 t - r\cos t + r\cos^2 t$

$= r - r\cos t$

$1-(-1) = 2$

17) $G(r,\theta) = (r\cos\theta,\ r\sin\theta)$

$\cos\theta \quad -r\sin\theta$

$\sin\theta \quad r\cos\theta$

$r\cos^2\theta + r\sin^2\theta$

$= 2r$

$= 8$

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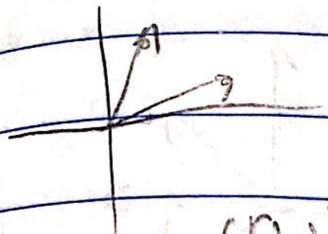
$(0,1) \to (2,3)$

$[0,1] \times [0,1]$ $(0,1)$,

to $\langle 2,3 \rangle$ and $\langle 4,1 \rangle$



$(0,1) \to (2,3)$

$(1,0) \to (4,1)$

$\cancel{(0,1) + (Au + Bv) = (2,3)}$

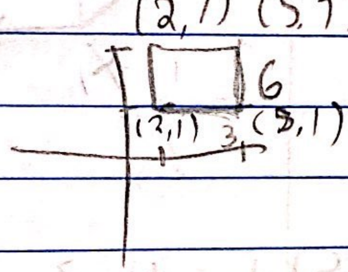
$\phi(u,v) = (Au + Bv,\ Cu + Dv)$

$\phi(0,1) = (2,3)$

$\phi(1,0) = (4,1)$

$\phi = (4u + 2v,\ u + 3v)$

3) $G(u,v) = (3u + v,\ u - 2v)$



$3 \quad 1$

$1 \quad -2$

$-6 - 1 = -7$

a) A: $(0,3) \times (0,5) = 15$

$15(7) = 105$

b) $(2,5) \times (1,7)$ area: 18

$18(7) = 126$

11/7/20 16.1) Vector Integrals

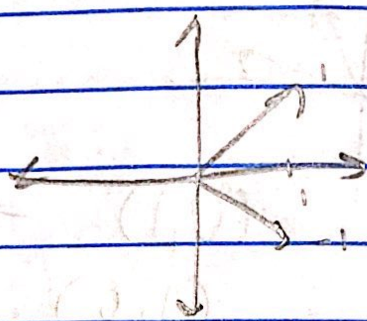
16.1) # 1, 3, 5, 7, 9, 11, 17, 23, 25, 27

1) $P=(1,2)$ Vector Field $F=\langle x^2, x\rangle$

$Q=(-1,-1)$

$F(P)= \langle 1,1\rangle$

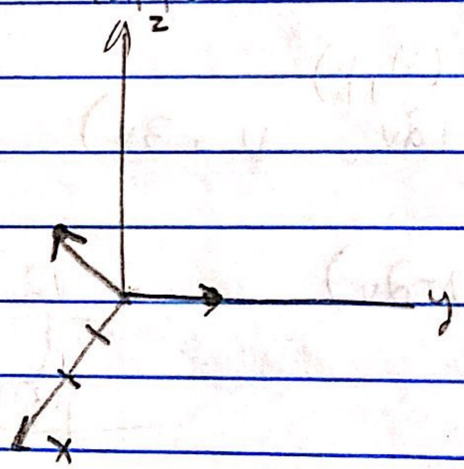
$F(Q)= \langle 1,-1\rangle$



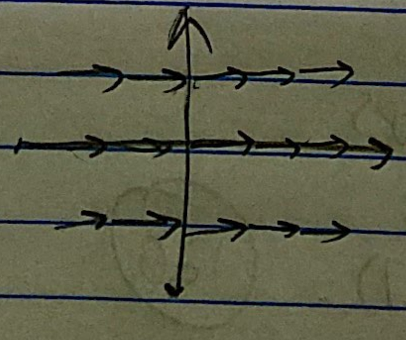
3) $F=\langle xy, z^2, x\rangle$

$P=(0,1,1)$ $F(P)= \langle 0,1,0\rangle$

$Q=(2,1,0)$ $F(Q)= \langle 2,0,2\rangle$



5) $F=\langle 1,0\rangle$

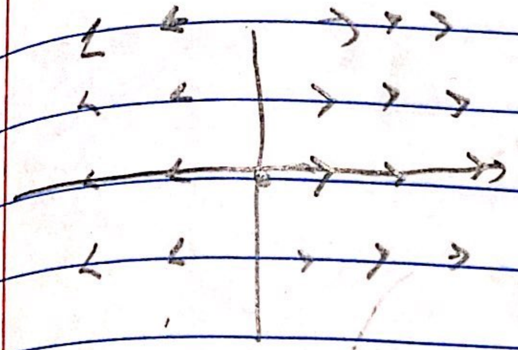


7)

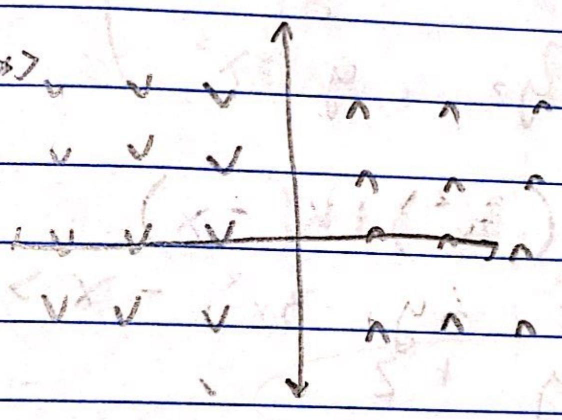
9)

11)

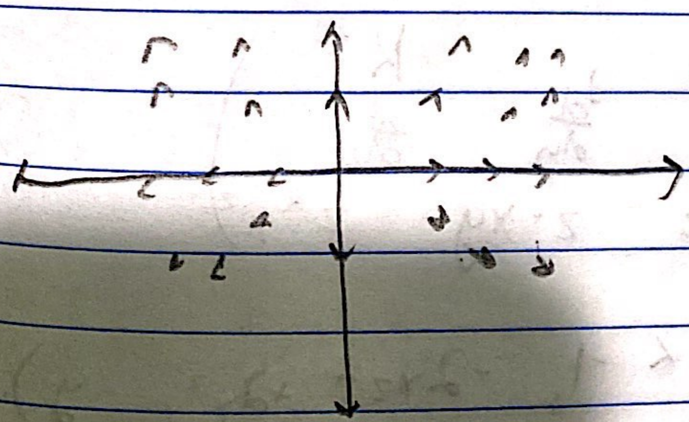
7) $F = \langle x, 0 \rangle \qquad r = \sqrt{x^2} = x$



9) $F = \langle 0, x \rangle$



11) $F = \left\langle \dfrac{x}{x^2+y^2}, \dfrac{y}{x^2+y^2} \right\rangle$



17) $F = \langle 1,1,1 \rangle \to C$

23) $F = \langle xy,\ yz,\ y^2 - x^3 \rangle$

$$\begin{pmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ xy & yz & y^2 - x^3 \end{pmatrix}$$

$i(y) - j((3x^2) + k(-x)$

$\text{curl}(F) = \langle y, -3x^2, -x \rangle$

$\text{div}(F) = y + z$

25) $F = \langle x - 2x^2 z,\ z - xy,\ z^2 x^2 \rangle$

$\text{div}(F) = \langle 1 - 4xz,\ -x,\ 2x^2 z \rangle$

$\text{curl}(F) =$

$$\begin{pmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x - 2x^2 z & z - xy & z^2 x^2 \end{pmatrix}$$

$= \langle -1,\ -2xz^2 + 2x^2,\ -y \rangle$

27) $F = \langle e^y, \sin x, \cos x \rangle$

$\operatorname{div}(F) = \langle 0, 0, 0 \rangle$

$$\begin{array}{ccc} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ e^y & \sin x & \cos x \end{array}$$

$= \langle 0, \sin x, -e^y + \cos x \rangle$

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