15.6) 1, 3, 13, 15, 17, 19, 23

1) $G(u,v)=(2u,\ u+v)$

a) The u- and v axes

$G(u,0)=(2u,u)$

$x=2u \quad y=u$

$u=\frac{x}{2} \quad y=\frac{x}{2}$

$G(0,v)=(0,v)$

$y=v$

b) The rectangle $R=[0,5]\times[0,7]$

$G(0,0)=(0,0)$
$G(5,0)=(10,5)$
$G(5,7)=(10,12)$
$G(0,7)=(0,7)$

parallelogram with vertices

c) $\vec{v}=\langle 4,1\rangle$ line seg

$G(4,1)=\langle 8,5\rangle$

line segment

d) Triangle $(0,1),(1,0),(1,1)$

$G(0,1)=(0,1)$
$G(1,0)=(2,1)$
$G(1,1)=(2,3)$

Triangle in xy-plane with vertices

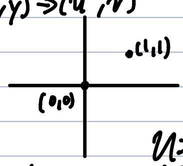
3) $G(u,v)=(u^2,v)$; not one to one; $\{(u,v),\ u\geq 0\}$ xor $\{(u,v),\ u\leq 0\}$

a) The u and v axes
$G(u,0)=(u^2,0)$ = x-axis
$G(0,v)=(0,v)$ = y axis

b) Rectangle $R=[-1,1]\times[-1,1]$
$0\leq u^2\leq 1 \quad -1\leq v\leq 1$
$0\leq u\leq 1 \quad -1\leq v\leq 1$ Rect $R'=[0,1]\times[-1,1]$

c) $(x,y)\Rightarrow(u^2,v)$

$x=u^2$
$v=y$



$u=v$
$\sqrt{x}=y$

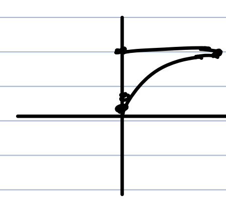
$0\leq u\leq 1$
$0\leq \sqrt{x}\leq 1$

$\boxed{\begin{gathered}0\leq y\leq\sqrt{x}\\ 0\leq x\leq 1\end{gathered}}$

d) $(0,0),(0,1),(1,1)$: Triangle

$G(0,0)=(0,0)$
$G(0,1)=(0,1)$
$G(1,0)=(1,0)$

$y=\sqrt{x} \quad 0\leq x\leq 1$



13) $G(u,v)=(3u+4v,\ u-2v)$

$x=3u+4v \quad y=u-2v$

$x_u=3 \quad y_u=1$
$x_v=4 \quad y_v=-2$ $=-6-4=-10$

15) $G(r,t)=(r\sin t,\ r-\cos t)$, $(r,t)=(1,\pi)$

$x=r\sin t$, $y=r-\cos t$

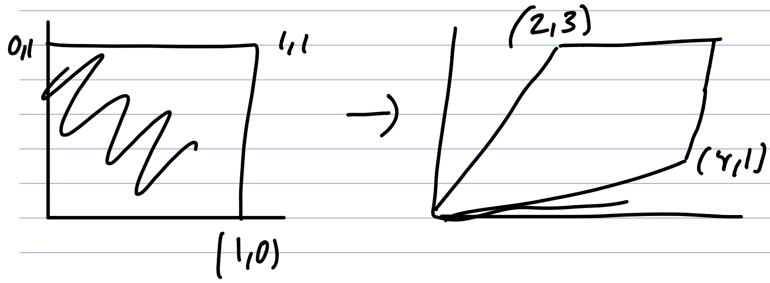
$\begin{vmatrix} x_r=\sin t & y_r=1 \\ x_t=r\cos t & y_t=\sin t\end{vmatrix}=\sin^2 t-r\cos t\Big|_{1,\pi}=1$

17) $G(r,\theta) = (r\cos\theta, r\sin\theta)$, $(r,\theta) = (4, \pi/6)$

$x = r\cos\theta \qquad y = r\sin\theta$

$$\begin{vmatrix} x_r = \cos\theta & y_r = \sin\theta \\ x_\theta = -r\sin\theta & y_\theta = r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta \Big|_4 = r\Big|_4 = 4$$

19) Find a linear mapping G that maps $[0,1] \times [0,1]$ to the parallelogram in the xy-plane spanned by vectors $\langle 2,3\rangle$ and $\langle 4,1\rangle$



$G(1,0) = \langle 4,1\rangle$ and $G(0,1) = \langle 2,3\rangle$

$G(1,0) = (A,B) \Rightarrow (4,1) \qquad G(0,1) = (C,D) \Rightarrow \langle 2,3\rangle$

$A=4 \quad B=1 \qquad C=2 \quad D=3$

$G(u,v) = (4u+v,\ 2u+3v)$

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23) $G(u,v) = (3u+v,\ u-2v)$

a) $x = 3u+v \qquad y = u-2v$

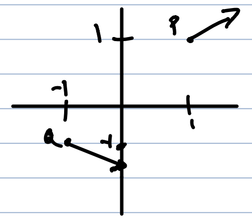
$$\begin{vmatrix} x_u = 3 & y_u = 1 \\ x_v = 1 & y_v = -2 \end{vmatrix} = -7$$

$$\int_0^5 \int_0^3 |-7| \, dv\, du = 105 \text{ square units}$$

b) $\int_1^7 \int_2^5 |-7| \, du\, dv = 126$ square units

16.1 [1, 3, 5, 7, 9, 11, 17, 23, 25, 27]

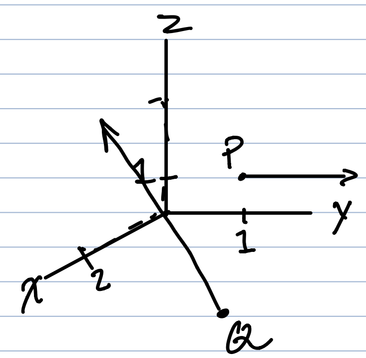
1) $P=(1,2)$ $Q=(-1,-1)$ $\vec{F}=\langle x^2, x\rangle$



$\vec{F}(P)=\langle 1,1\rangle$
$\vec{F}(Q)=\langle 1,-1$

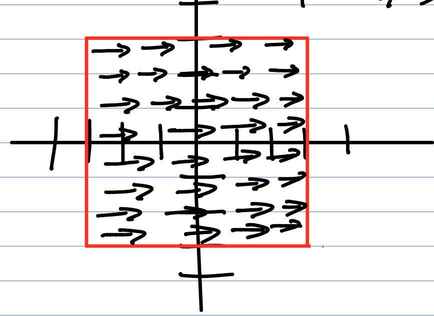
3) $P=(0,1,1)$ $Q=(2,1,0)$ $\vec{F}=\langle xy, z^2, x\rangle$

$\vec{F}(P)=\langle 0,1,0\rangle$
$\vec{F}(Q)=\langle 2,0,2\rangle$

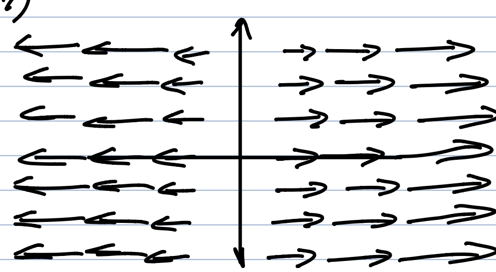


5) $-3\le x\le 3$ $-3\le y\le 3$

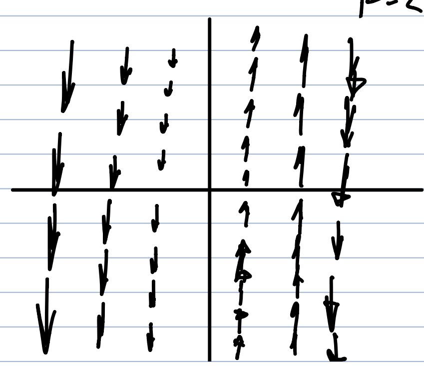
$F=\langle 1,0\rangle$



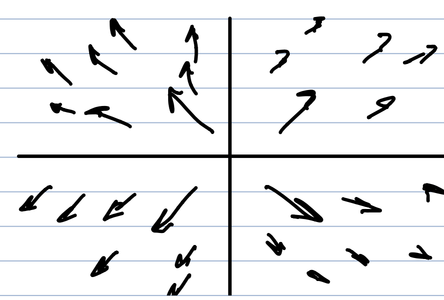
7) $F=x\,i$



9) $F=\langle 0, x\rangle$



11) $F=\left\langle \dfrac{x}{x^2+y^2}, \dfrac{y}{x^2+y^2}\right\rangle$



17) $\vec{F} = \langle 1, 1, 1 \rangle$

C

23) $F = \langle xy, yz, y^2 - x^3 \rangle$

$\mathrm{div}(F) = \nabla \cdot F = \left\langle \frac{\delta}{\delta x}, \frac{\delta}{\delta y}, \frac{\delta}{\delta z} \right\rangle \cdot \langle xy, yz, y^2 - x^3 \rangle$

$= \frac{\delta}{\delta x} xy + \frac{\delta}{\delta y} yz + \frac{\delta}{\delta z} y^2 x^3 = y + z$

$\mathrm{curl} = \nabla \times F = \left\langle \frac{\delta}{\delta x}, \frac{\delta}{\delta y}, \frac{\delta}{\delta z} \right\rangle \times \langle xy, yz, y^2 - x^3 \rangle$

$= \left(\frac{\delta}{\delta y} y^2 - x^3 - \frac{\delta}{\delta z} yz \right) \hat{i} + \left(\frac{\delta}{\delta z} xy - \frac{\delta}{\delta x} (y^2 - x^3) \right) \hat{j} + \left(\frac{\delta}{\delta x} yz - \frac{\delta}{\delta y} xy \right) \hat{k}$

$= y\hat{i} + 3x^2)\hat{j} + (-x)\hat{k}$

25) $\vec{F} = \langle x - 2zx^2, z - xy, z^2 x^2 \rangle$

*Used maple because its the same process

$\mathrm{div}(\vec{F}) = 1 - 4xz - x + 2x^2 z$

$\mathrm{Curl}(\vec{F}) = -\hat{i} + 2x^2 - 2xz^2 \hat{j} - y\vec{k}$

27) $\vec{F} = \langle z - y^2, x + z^3, y + x^2 \rangle$

*Used maple because its the same process

$\mathrm{div}(\vec{F}) = 0$

$\mathrm{Curl}(\vec{F}) = (1 - 3z^2)i + (1 - 2x)\hat{j} + (1 + 2y)\hat{k}$