15.6

1. (a) on u-axis $v=0$; on v-axis $u=0$

image set: $\{(2u,u),(0,v)\}$

image set: $\{(u,v),(2u,u),(0,v)\}$

(b) $R=[0,5]\times[0,7]$

$G(0,0)=(0,0)$, $G(5,0)=(10,5)$

$G(0,7)=(0,7)$ $G(5,7)=(10,12)$

(c). $\frac{u-1}{2-1}=\frac{v-5}{3-5}$

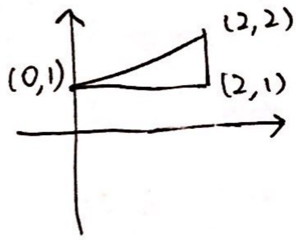
$v-5=-2(u-1)$

$2u+v=7$

$2(2u)+(u+v)=7$

$5u+v=7$

(d).



3. (a). image of u-axis

$(u,0)$

image of v-axis

$(0,v)$

(b) ∵ $|u|\le 1, |v|\le 1$

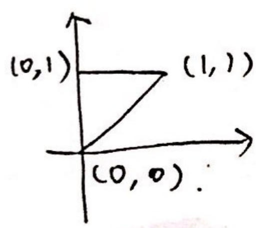
$x=u^2, y=v \Rightarrow u=\pm\sqrt{x}, v=y$

$|\sqrt{x}|\le 1$, $|y|\le 1$

(c) $0\le u\le 1$, $v=u$

$0\le\sqrt{x}\le 1$, $y=\sqrt{x}$

(d).



13. $G(u,v) = (3u+4v;\ u-2v)$

$$J = \begin{vmatrix} \frac{d(3u+4v)}{du} & \frac{d(3u+4v)}{dv} \\ \frac{d(u-2v)}{du} & \frac{d(u-2v)}{dv} \end{vmatrix}$$

$= 3\times(-2) - (1)(4)$

$= -10$

15. $G(r,t) = (r\sin t,\ r-\cos t) \quad (r,t) = (1,\pi)$

$$J = \begin{vmatrix} \frac{d(r\sin t)}{\cancel{dt}\,dr} & \frac{d(r\sin t)}{dt} \\ \frac{d(r-\cos t)}{\cancel{dt}\,dr} & \frac{d(r-\cos t)}{dt} \end{vmatrix}$$

$= \sin(t)\cdot\sin(t) - \cancel{\sin(t)}\ r\cos(t)\cdot 1$

$(r,t) = (1,\pi)$

$= 0 - (-1)$

$= 1$

17. $G(r,\theta) = (r\cos\theta,\ r\sin\theta),\ (r,\theta) = (4,\frac{\pi}{6})$

$$J = \begin{vmatrix} \frac{d(r\cos\theta)}{dr} & \frac{d(r\cos\theta)}{d\theta} \\ \frac{d(r\sin\theta)}{dr} & \frac{d(r\sin\theta)}{d\theta} \end{vmatrix}$$

$= r\sin^2(\theta) + r\cos^2(\theta)$

$(r,\theta) = (4,\frac{\pi}{6})$

$= 4\cdot(\frac{1}{2})^2 + 4\cdot(\frac{\sqrt{3}}{2})^2$

$= 1+3$

$= 4$

19. Let $\phi(u,v)$ be $=(Au+Bv, Cu+Dv)$

$\phi(0,1) = (2,3)$

$\phi(0,1) = (B,D) = (2,3)$

$\phi(0,1) = (4,1) = (A,C)$

$\phi(u,v) = (4u+2v, u+3v)$

23. $G(u,v) = (3u+v, u-2v)$

$J = \begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} = -7$

So the area is 7

(a) $R = [0,3] \times [0,5]$

which is $3 \times 5 = 15$

$\text{Area}(\phi(R)) = 7 \times 15 = 105$

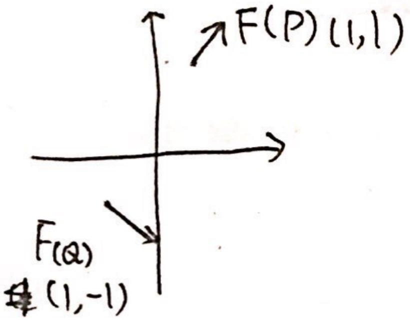
(b) $R = [2,5] \times [1,7]$

which is $3 \times 6 = 18$

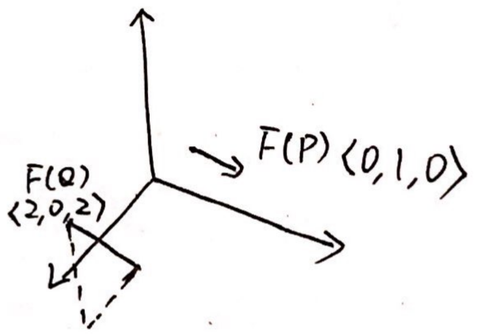
$\text{Area}(\phi(R)) = 7 \times 18 = 126$

16.1

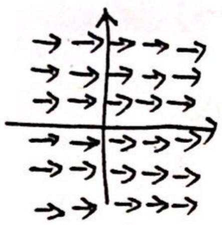
1. $F(1,2)=\langle 1,2\rangle$ $F(-1,-1)=\langle 1,-1\rangle$



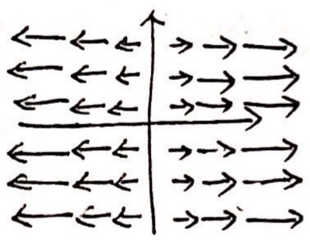
3. $F(0,1,1)=\langle 0,1,0\rangle$, $F(2,1,0)=\langle 2,0,2\rangle$



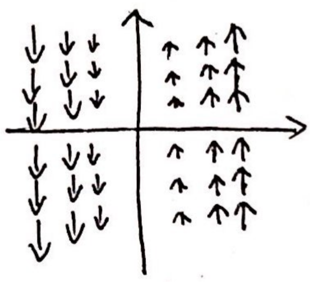
5. $F=\langle 1,0\rangle$



7. $F=xi$



9. $F=\langle 0,x\rangle$



11. $F=\left\langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}\right\rangle$

17. $F = \langle 1, 1, 1 \rangle$

Picture C

23. $F = \langle xy, yz, y^2 - x^3 \rangle$

$\operatorname{div} F = \frac{d}{dx}(xy) + \frac{d}{dy}(yz) + \frac{d}{dz}(y^2 - x^3)$

$= y + z$

$\operatorname{curl} F = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ xy & yz & y^2 - x^3 \end{vmatrix}$

$= y\,i - 3x^2 j + (-x) k$

25. $F = \langle x - 2zx^2, z - xy, z^2 x^2 \rangle$

$\operatorname{div} F = 2x^2 z - x(4z+1) + 1$

$\operatorname{curl} F = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x - 2zx^2 & z - xy & z^2 x^2 \end{vmatrix}$

$= -i - 2x(x + z^2) j - y k$

27. $F = \langle z - y^2, x^2 + z^3, y + x^2 \rangle$

$\operatorname{div} F = 0$

$\operatorname{curl} F = (1 - 3z^2) i - (1 - 2x) j + (2y + 1) k$