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15.6: 1, 3, 13, 15, 17, 19, 23

#1 $G(u,v) = (2u, u+v)$

a) u-axis: $(u,0) \rightarrow (2u, 0)$

$x = 2u \longrightarrow u = \frac{x}{2}$

v-axis: $(0,v) \rightarrow (0, 0+v)$

$y = v$

b) R': $(0,0) \rightarrow (2(0), 0+0) = (0,0)$

$(0,7) \rightarrow (2(0), 0+7) = (0,7)$

$(5,0) \rightarrow (2(5), 5+0) = (10,5)$

$(5,7) \rightarrow (2(5), 5+7) = (10,12)$

c) $(1,2) \rightarrow (2(1), 1+2) = (2,3)$

$(5,3) \rightarrow (2(5), 5+3) = (10,8)$

line segment joining (2,3) and (10,8)

d) $(0,1) \rightarrow (2(0), 0+1) = (0,1)$

$(1,0) \rightarrow (2(1), 1+0) = (2,1)$

$(1,1) \rightarrow (2(1), 1+1) = (2,2)$

triangle with vertices (0,1) (2,1), (2,

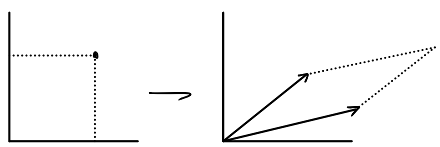
#17 $G(r,\theta) = (r\cos\theta, r\sin\theta)$ $(r,\theta) = (4, \frac{\pi}{6})$

$J = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta$

$\rightarrow r(\cos^2\theta + \sin^2\theta = r(1)$

$= \boxed{4}$

#19



$G(u,v) = (Au + Bv, Cu + Dv)$

$(0,1) = (A, C) = (2,3)$

$(0,1) = (B, D) = (4,1)$

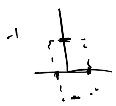
$\boxed{G(u,v) = (2u + 4v, 3u + v)}$

#3 $G(u,v) = (u^2, v)$

a) G is not one to one

u axis: $(u,0) \rightarrow x = u^2 \quad y = 0$

v axis: $(0,v) \rightarrow x = 0 \quad y = v$



b) R': $(-1,1) \rightarrow (1,1)$

$(-1,-1) \rightarrow (1,-1)$

$(1,1) \rightarrow (1,1)$

$(1,-1) \rightarrow (1,-1)$

c) $(0,0) \rightarrow (0,0)$

$(1,1) \rightarrow (1,1)$

d) $(0,0) \rightarrow (0,0)$

$(0,1) \rightarrow (0,1)$

$(1,1) \rightarrow (1,1)$

#13 $G(u,v) = (3u + 4v, u - 2v)$

$J: \begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} = 3(-2) - 4(1) = \boxed{-10}$

#15 $G(r,t) = (r\sin t, r - \cos t)$, $(r,t) = (1, \pi)$

$J: \begin{vmatrix} \sin t & r\cos t \\ 1 & \sin t \end{vmatrix} = \sin t(\sin t) - r\cos t$

$= \sin^2 t - r\cos t \longrightarrow \sin^2(\pi) - \cos(\pi)$

$= \boxed{1}$

#23 $G(u,v) = (3u + v, u - 2v)$

$J = \begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} = -6 - 1 = -7$

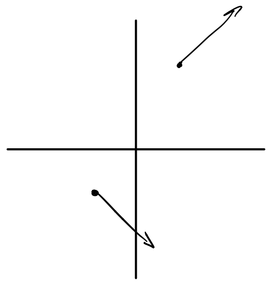
a) $R = [0,3] \times [0,5] \longrightarrow |-7| \cdot 15 = \boxed{105}$

b) $R = [2,5] \times [1,7] \longrightarrow |-7| \cdot 18 = \boxed{126}$

16.1 : 1, 3, 5, 7, 9, 11, 17, 23, 25, 27

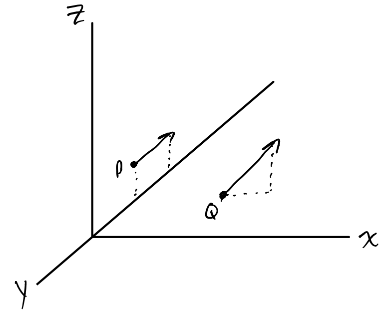
#1 $P = (1,2) \to F = (1,2)$

$Q = (-1,-1) \to F = (1,-1)$

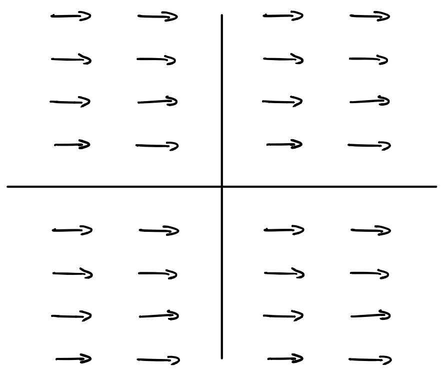


#3 $P = (0,1,1) \to F = \langle 0(1), 1^2, 0 \rangle = \langle 0,1,0 \rangle$

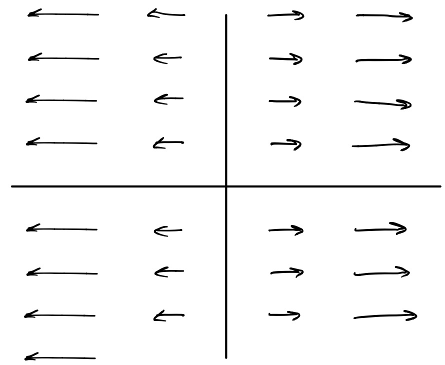
$Q = (2,1,0) \to F = \langle 2(1), 0^2, 2 \rangle = \langle 2,0,2 \rangle$



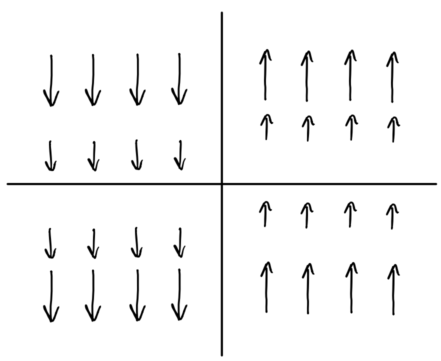
#5 $F = \langle 1,0 \rangle$



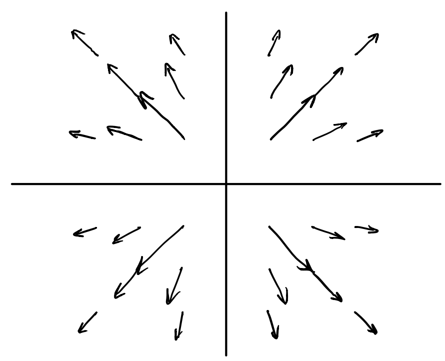
#7 $F = x\mathbf{i} \to \langle x, 0 \rangle$



#9 $F = \langle 0, x \rangle$



#11 $F = \left\langle \dfrac{x}{x^2+y^2}, \dfrac{y}{x^2+y^2} \right\rangle$



$\frac{1}{1+4}$ $\quad \frac{2}{5}$

$\frac{1}{5}$,

#17 C

#23 $F = \langle xy, yz, y^2 - x^3 \rangle$

$\text{div} F = y + z + 0 = \boxed{y+z}$

$\text{curl} F = \langle 2y - y, 0 + 3x^2, 0 - x \rangle$

$= \boxed{\langle y, 3x^2, -x \rangle}$

#25 $F = \langle x - 2zx^2, z - xy, z^2x^2 \rangle$

$\text{div} F = (1 - 4zx) + (-x) + (2x^2z)$

$= \boxed{1 - 4zx - x + 2x^2z}$

$\text{curl} F = \langle 0 - 1, -2x^2 - 2z^2x, -y - 0 \rangle$

$= \boxed{\langle -1, -2x^2 - 2z^2x, -y \rangle}$

#27 $F = \langle z - y^2, x + z^3, y + x^3 \rangle$ $\quad \text{curl} F = \boxed{\langle 1 - 3z^2, 1 - 3x^2, 1 + 2y \rangle}$

$\text{div} F = 0$