15.6

1. a. $x = 2u$ $\quad y = u + v$ $\quad y = \cancel{\frac{x}{2}}^{0} + v$

$\frac{x}{2} = u$ $\quad y = \frac{1}{2}x + \cancel{v}^{0}$ $\quad y = v$

$y = \frac{1}{2}x$ u axis $\quad$ v axis

b. $G(0,0) = (0,0)$

$G(0,7) = (0,7)$

$G(5,0) = (10,5)$

$G(5,7) = (10,2)$

c. $G(1,2) = (2,3)$

$G(5,3) = (10,8)$

d. $G(0,1) = (0,1)$

$G(1,0) = (2,1)$

$G(1,1) = (2,2)$

3. $G(u,v) = (u^2, v)$

a. $x = u^2$ $\quad y = v$ $\quad$ Not 1-to-1

$x = 0$ $\quad y = 0$

y axis $\quad$ x axis $\quad$ $u \neq 0 \rightarrow$ Domain

b. $G(-1,-1) = (1,1)$

$G(1,-1) = (1,-1)$

$G(-1,1) = (1,1)$

$G(1,1) = (1,1)$ $\quad$ Rectangle $[0,1] \times [-1,1]$

c. $(0,0)$ to $(1,1)$

$m = 1$ $\quad y = \sqrt{x}$ $\quad 0 \leq x \leq 1$

d. $(0,0)\ (0,1)\ (1,1)$

$G(0,0) = (0,0)$

$G(0,1) = (0,1)$

$G(1,1) = (1,1)$

13. $G(u,v) = (3u+4v,\ u-2v)$

$x = 3u+4v \qquad y = v - 2v$

$\frac{dx}{du} = 3 \qquad \frac{dx}{dv} = 4 \qquad \frac{dy}{du} = 1 \qquad \frac{dy}{dv} = -2$

$$\begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} = -6 - 4 = -10$$

15. $G(r,t) = (r\sin t,\ r - \cos t) \qquad (r,t) = (1, \pi)$

$x = r\sin t \qquad y = r - \cos t$

$\frac{dx}{dr} = \sin t \qquad \frac{dx}{dt} = r\cos t \qquad \frac{dy}{dr} = 1 \qquad \frac{dy}{dt} = \sin t$

$= \sin\pi = 0 \qquad = 1\cos\pi = -1 \qquad = 1 \qquad = \sin\pi = 0$

$$\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = 0 + 1 = 1$$

17. $G(r,\theta) = (r\cos\theta,\ r\sin\theta) \quad (r,\theta) = (4, \frac{\pi}{6})$

$\frac{dx}{dr} = \cos\theta \qquad \frac{dx}{d\theta} = -r\sin\theta \qquad \frac{dy}{dr} = \sin\theta \qquad \frac{dy}{d\theta} = r\cos\theta$

$= \cos\frac{\pi}{6} \qquad = -4\sin\frac{\pi}{6} \qquad = \sin\frac{\pi}{6} \qquad = 4\cos\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2} \qquad = -2 \qquad = \frac{1}{2} \qquad = \frac{4\sqrt{3}}{2}$

$$\begin{vmatrix} \frac{\sqrt{3}}{2} & -2 \\ \frac{1}{2} & \frac{4\sqrt{3}}{2} \end{vmatrix} = \frac{12}{4} + 1 = 4$$

19. $x = au + bv \qquad y = cu + dv \qquad \langle 2,3 \rangle \quad \langle 4,1 \rangle$

$x = 4u + 2v \qquad y = 1u + 3v$

23. $\frac{dx}{du} = 3 \qquad \frac{dx}{dv} = 1 \qquad \frac{dy}{du} = 1 \qquad \frac{dy}{dv} = -2 \qquad \begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} = -6 - 1 = -7$

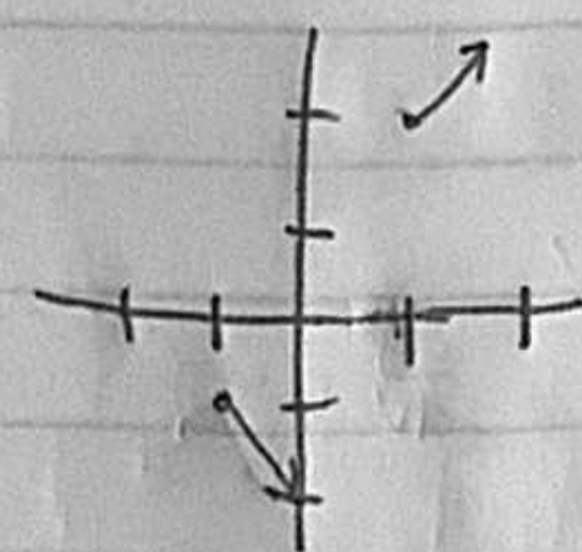
a. $|-7| \cdot (3 \cdot 5) = 105$

b. $\int_2^5 \int_1^7 |-7|\, dv\, du = 7v \Big|_1^7 = \int_2^5 42\, du = 42u \Big|_2^5 = 210 - 84 = 126$

16.1

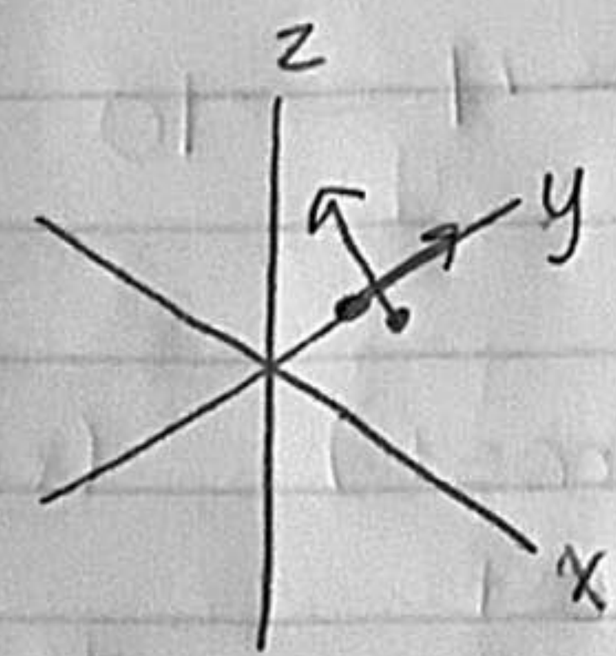
1. $\langle x^2, x\rangle$

$\langle 1, 2\rangle \quad \langle 1, -1\rangle$

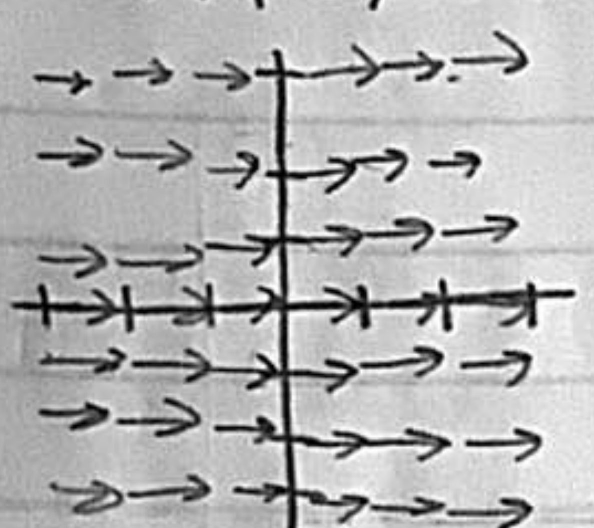


3. $\langle xy, z^2, x\rangle$

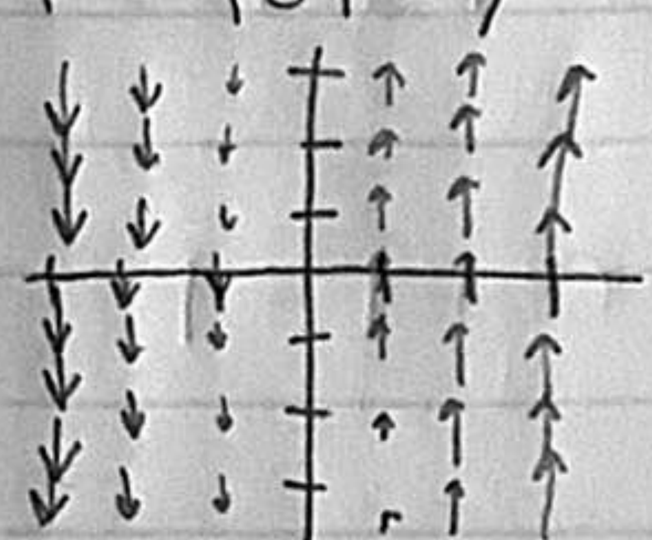
$\langle 0, 1, 0\rangle \quad \langle 2, 0, 2\rangle$



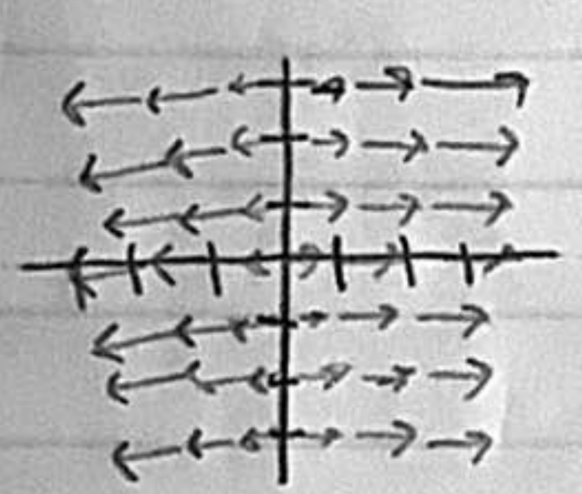
5. $F = \langle 1, 0\rangle$



9. $F = \langle 0, x\rangle$



7. $F = x\,i$



11. $F = \left\langle \frac{x}{x^2+y^2}, \frac{x}{x^2+y^2}\right\rangle$

x	y	F
-3	-3	$\langle -\frac{1}{6}, -\frac{1}{6}\rangle$
-2	-2	$\langle -\frac{1}{2}, -\frac{1}{2}\rangle$
-1	-1	$\langle -1, -1\rangle$
0	0	$\langle 0, 0\rangle$
1	1	$\langle 1, 1\rangle$
2	2	$\langle \frac{1}{2}, \frac{1}{2}\rangle$
3	3	$\langle \frac{1}{6}, \frac{1}{6}\rangle$

17. C

23. $F = \langle xy, yz, y^2 - x^3\rangle$

$$\frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial y}(yz) + \frac{\partial}{\partial z}(y^2 - x^3) = x + y$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & y^2-x^3 \end{vmatrix} = (2y - y)\,i - (-3x^2)\,j + -(x)\,k$$

$$= \langle y, 3x^2, -x\rangle$$

25. $\operatorname{div}(F) = \langle 1 - 4zx, -x, 2zx^2 \rangle$

$$\operatorname{curl}(F) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x - 2zx^2 & z - xy & z^2x^2 \end{vmatrix} =$$

$$= \langle -1, 2x^2 - 2xz^2, -y \rangle$$

27. $\operatorname{div}(F) = \langle 0, 0, 0 \rangle$

$$\operatorname{curl}(F) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z - y^2 & x + z^3 & y + x^2 \end{vmatrix} =$$

$$= \langle 1 - 3z^2, 1 - 2x, 1 + 2y \rangle$$