15.6 Homework

① $G(u,v) = (2u, u+v)$

→ a) u-axis: $y = \frac{1}{2}x$, v-axis: $y = 0$

→ b) Parallelogram $[(0,0), (10,5), (10,2), (0,7)]$

→ c) Line joining $(2,3)$ and $(10,8)$

→ d) Triangle $[(0,1), (2,1)]$

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③ $G(u,v) = (u^2, v)$

→ a) pos. x-axis, neg. y-axis

→ b) $R = [0,1] \times [-1,1]$

→ c) $y = \sqrt{x}$ with interval $0 \leq x \leq 1$

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⑬ $G(u,v) = (3u+4v, u-2v)$

→ $x = 3u+4v$, $y = u-2v$

→ -10

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⑮ $G(r,t) = (r\sin t, r - \cos t)$, $(r,t) = (1, \pi)$

→ $x = r\sin t$, $y = r - \cos t$

→ $Jac(G) = \sin^2 t - r\cos t = \sin^2 \pi - \cos \pi = 1$

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⑰ $G(r,\theta) = (r\cos\theta, r\sin\theta)$, $(r,\theta) = (4, \frac{\pi}{6})$

→ $x = r\cos\theta$, $y = r\sin\theta$

→ $Jac(G) = -r\sin^2\theta - r\cos^2\theta$

→ $Jac(G) = 4\sin^2\frac{\pi}{6} + 4\cos^2\frac{\pi}{6} = 4$

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(19) $R = [0,1] \times [1,0]$, $\langle -2,5 \rangle$, $\langle 4,1 \rangle$

→ $G(u,v) = (au + bv,\ cu + dv)$

→ $G(u,v) = (4u + 2v,\ u + 3v)$

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(23) $G(u,v) = (3u + v,\ u - 2v)$

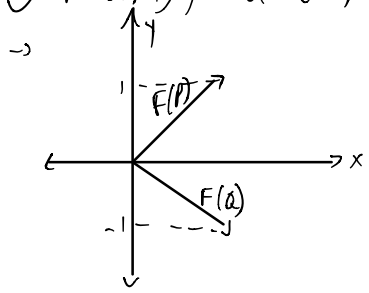
→ $\text{Jac}(G) = -7$

→ a) Area = 105

→ b) Area = 126

16.1 Homework

① $P=(1,2)$, $Q=(-1,-1)$, $F=\langle x^2, x\rangle$, $F(P)=\langle 1,1\rangle$, $F(Q)=\langle 1,-1\rangle$



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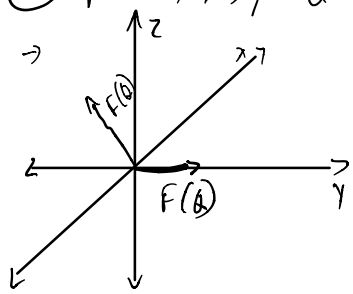
③ $P=(0,1,1)$, $Q=(2,1,0)$, $F=\langle xy, z^2, x\rangle$, $F(P)=\langle 0,1,0\rangle$, $F(Q)=\langle 2,0,2\rangle$



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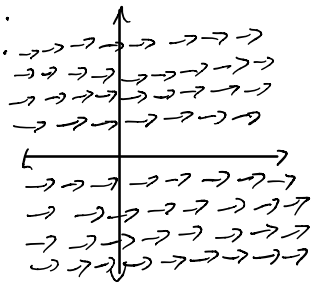
⑤ Graph:



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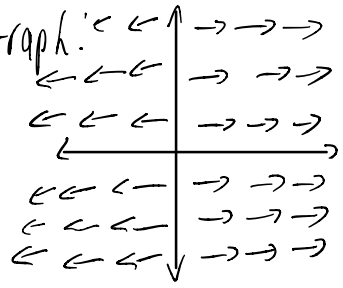
⑦ $F=x$, $F=\langle x, 0\rangle$

Graph:



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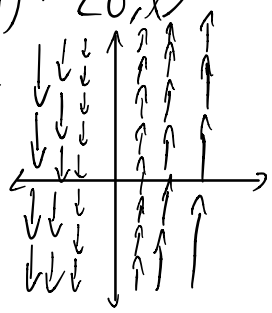
(9) $F(x,y) = \langle 0, x \rangle$

→ Graph:



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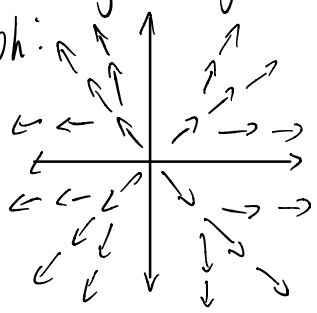
(11) $F = \left\langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right\rangle$

→ Graph:



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(17) $F = \langle 1, 1, 1 \rangle$ (C.)

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(23) $F = \langle xy, yz, y^2 \cdot x^3 \rangle$

→ $\begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ xy & yz & y^2 \cdot x^3 \end{vmatrix}$ so $curl(F) = \langle y, 3x^2, -x \rangle$

→ $\frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz}$ so $div(F) = y + z$

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(25) $F = \langle x - 2zx^2, z - xy, z^2 x^2 \rangle$

→ $\begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ x - 2zx^2 & z - xy & x^2 z^2 \end{vmatrix}$ so $curl(F) = \langle -1, -2x(x+z^2), -y \rangle$

→ $\frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz}$ so $div(F) = 2x^2 z - x(4x+1) + 1$

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(27) $F = \langle 2-y^2,\ x+z^3,\ z^2x^2 \rangle$

$\rightarrow \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ 2-y^2 & x+z^3 & z^2x^2 \end{vmatrix}$ so $\text{curl}(F) = \langle -3z^2,\ 1-2xz^2,\ 2y+1 \rangle$

$\rightarrow \frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz}$ so $\text{div}(F) = 2x^2z$
