

15.6

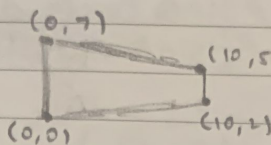
1a) u-axis

v-axis

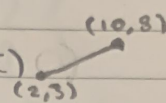
$$y = \frac{1}{2}x$$

y-axis

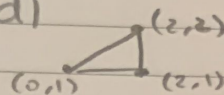
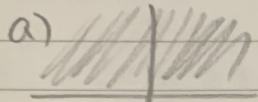
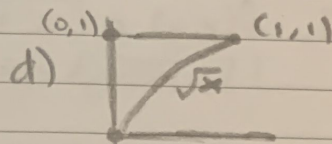
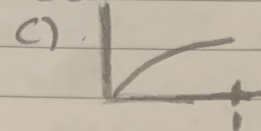
b)



c)



d)

3.)  $G$  is not one to one,  $u \geq 0$  and  $u \leq 0$ b)  $[0,1] \times [-1,1]$ 

$$13. \quad G(u,v) = (3u+4v, u-2v) \quad \begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} = \boxed{-10}$$

$$15. \quad G(r,t) = (r \sin t, r \cos t), \quad (r,t) = (1, \pi)$$

$$\begin{vmatrix} \sin t & r \cos t \\ 1 & \sin t \end{vmatrix} = (\sin^2 t) - r \cos t = \boxed{1}$$

17.  $G(r, \theta) = (r \cos \theta, r \sin \theta), (r, \theta) = (4, \frac{\pi}{2})$

$$\begin{vmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{vmatrix} = r \cos^2 \theta + r \sin^2 \theta = r = \boxed{4}$$

19.  $[0, 1] \times [0, 1] \rightarrow \langle 4u+2v, u+3v \rangle$

23a)  $G(u, v) = (3u+v, u-2v) R = [0, 3] \times [0, 5]$

$$\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} = -6 - 1 = -7 \quad \iint_R (3u+v+u-2v) 7 dA$$

$7 \iint_R (4u-v) dA \rightarrow$  bounds are found using  $[0, 3] \times [0, 5]$

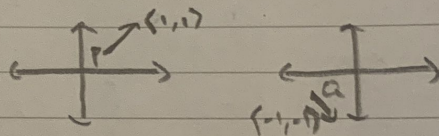
$G(R) = 7 \iint (4u-v) dA \rightarrow \boxed{105}$

b)  $\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} = -7 = \iint (3u+v+u-2v) 7 dA$

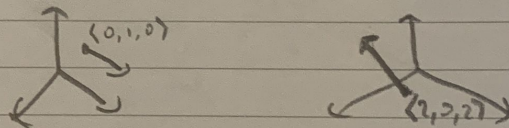
Same as a) with bounds being  $[2, 5] \times [1, 7] \rightarrow \boxed{126}$

16.1

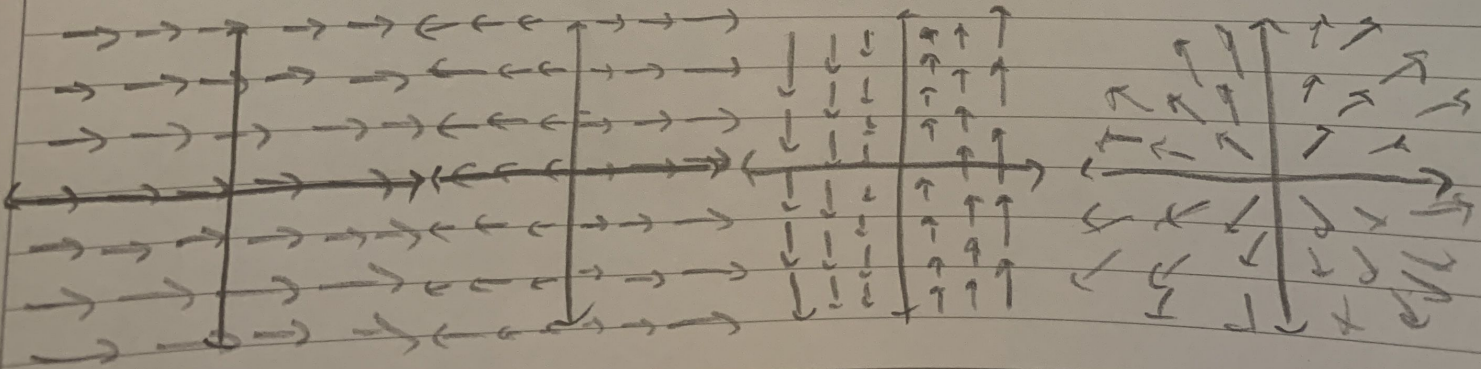
1.  $P = (1, 2) \quad Q = (-1, -1) \quad F = \langle x^2, x \rangle$   
 $F(1, 2) = \langle 1, 1 \rangle \quad F(-1, -1) = \langle 1, -1 \rangle$



3.  $P = (0, 1, 1) \quad Q = (2, 1, 0) \quad F = \langle xy, z^2, x \rangle$   
 $F(0, 1, 1) = \langle 0, 1, 0 \rangle \quad F(2, 1, 0) = \langle 2, 0, 2 \rangle$



5.  $F = \langle 1, 0 \rangle$     7.  $F = x_i$     9.  $F = \langle 0, x \rangle$     11.  $\langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \rangle$



17.  $F = \langle 1, 1, 1 \rangle$  Plot c

23.  $F = \langle xy, yz, y^2 - x^3 \rangle$   $\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & y^2 - x^3 \end{vmatrix}$

$$\nabla \times F = \left[ \frac{\partial}{\partial y}(y^2 - x^3) - \frac{\partial}{\partial z}(yz) \right] i - \left[ \frac{\partial}{\partial x}(y^2 - x^3) - \frac{\partial}{\partial z}(xy) \right] j - \left[ \frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial y}(xy) \right] k$$
$$\text{curl } F = \langle y, 3x^2, -x \rangle$$

$$\text{div } F = 1 - \text{div } F = y + z$$

25.  $F = \langle x - 2zx^2, z - xy, z^2x^2 \rangle$

$$\text{div } F = 1 - 4zx - x + 2x^2z$$

$$\text{curl } F = \nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x - 2zx^2 & z - xy & z^2x^2 \end{vmatrix} = \langle -1, 2x^2 - 2xz^2, -y \rangle$$

27.  $F = \langle z - y^2, x + z^3, y + x^2 \rangle$

$$\text{div } F = 0$$

$$\text{curl } F = \nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z - y^2 & x + z^3 & y + x^2 \end{vmatrix} = \langle 1 - 3z^2, 1 - 2x, 1 + 2y \rangle$$