15.6 HW

1. $G(u,v) \quad (2u, u+v)$

a. u-axis

$G(u,0) = (2u + u + 0)$

$= (2u, u)$

$y = x/2$

v-axis

$G(0,v) = (0,v)$

$x = 0$

y-axis

b. $G(0,0) = (0,0)$

$G(5,0) = (10,5)$

$G(5,7) = (10,12)$

$G(0,7) = (0,7)$

Rectangle with these vertices

c. $(1,2) \quad (5,3)$

$G(u,v) = (2u, u+v)$

$G(1,1) = (2,3)$

$G(5,1) = (10,1)$

Line segment btw $(2,3)$ $(10,8)$

d. $(0,1) \to (0,1)$

$(1,0) \to (2,1)$

$(1,1) \to (2,2)$

Triangle

3. a. $G(u,0) = (u^2, 0)$

$y = 0$

x-axis

b. $-1 \leq u \leq 1$

$0 \leq u^2 \leq 1$

$0 \leq x \leq 1$

$-1 \leq v \leq 1$

$-1 \leq y \leq 1$

Region $[0,1]$ $[-1,1]$

c. $(x, y) = (u^2, v)$

$u = v$ since $(0,0)$ $(1,1)$

so $u = \sqrt{x}$ $v = y$

$y = \sqrt{x}$ $0 \leq x \leq 1$

13. $G(u, v) = (3u + 4v, u - 2v)$

Jacobian(G)

$x = 3u + 4v$ $\frac{\partial x}{\partial u} = 3$ $\frac{\partial x}{\partial v} = 4$

$y = u - 2v$ $\frac{\partial y}{\partial u} = 1$ $\frac{\partial y}{\partial v} = -2$

$$\begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} = \boxed{-10}$$

15. $G(r, t) = (r\sin t, r - \cos t)$

$x = r\sin t$ $\frac{\partial x}{\partial r} = \sin t$ $\frac{\partial x}{\partial t} = r\cos t$

$y = r - \cos t$ $\frac{\partial y}{\partial r} = 1$ $\frac{\partial y}{\partial t} = \sin t$

$$\begin{vmatrix} \sin t & r\cos t \\ 1 & \sin t \end{vmatrix} = \sin^2 t - r\cos t$$

at $(1, \pi) = \sin^2 \pi - 1\cos \pi$

$= 0 - (-1)$

$= \boxed{1}$

17. $G(r,\theta) = (r\cos\theta, r\sin\theta)$ at $(4, \pi/6)$

$x = r\cos\theta$ $\quad \frac{\partial x}{\partial r} = \cos\theta \quad \frac{\partial x}{\partial \theta} = -r\sin\theta$

$y = r\sin\theta$ $\quad \frac{\partial y}{\partial r} = \sin\theta \quad \frac{\partial y}{\partial \theta} = r\cos\theta$

$$\begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$= r\cos^2\theta + r\sin^2\theta$

$= r(\sin^2\theta + \cos^2\theta)$

$= r$

at $(4, \pi/6)$

$= \boxed{4}$

19. ?

23. $G(u,v) = (3u+v, u-2v)$

a. $R: [0,3] \times [0,5]$

$x = 3u+v \quad \frac{\partial x}{\partial u} = 3 \quad \frac{\partial x}{\partial v} = 1$

$y = u-2v \quad \frac{\partial y}{\partial u} = 1 \quad \frac{\partial y}{\partial v} = -2$

Jacobian: $\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} = -6-1 = -7$

$$\int_0^5 \int_0^3 7 \, du \, dv = 105$$

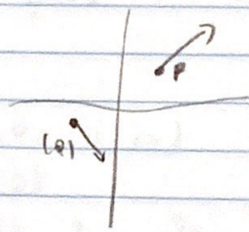
b. $$\int_1^7 \int_2^5 7 \, du \, dv = 126$$

16.1 HW

1. $F=\langle x^2, x\rangle$ $P=(1,2)$ $Q=(-1,-1)$

$F(P)=\langle 1,1\rangle$

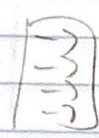
$F(Q)=\langle 1,-1\rangle$



3. $F=\langle xy, z^2, x\rangle$ $P=(0,1,1)$ $Q=(2,1,0)$

$F(P)=\langle 0,1,0\rangle$

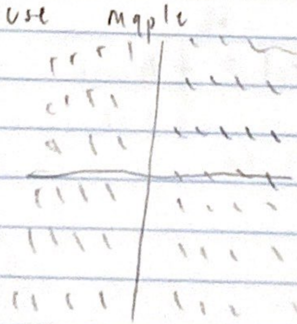
$F(Q)=\langle 2,0,2\rangle$



5. $F=\langle 1,0\rangle$

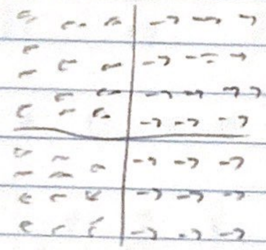
$-3\le x\le 3$ $-3\le y\le 3$

We can use maple



7. $F = x\hat{i}$

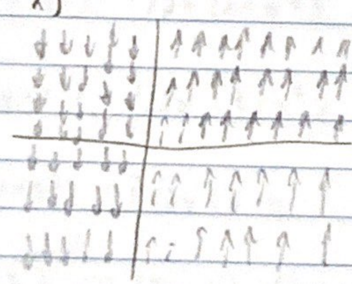
$-3 \leq x \leq 3$ $-3 \leq y \leq 3$ Just follow direction of x



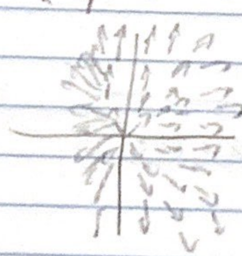
9. $F = \langle 0, x \rangle$

$-3 \leq x \leq 3$ $-3 \leq y \leq 3$

$x\hat{j}$



11. $F = \left\langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right\rangle$ $-3 \leq x \leq 3$ $-3 \leq y \leq 3$



17. C

23. $F = \langle xy, yz, y^2 - x^3 \rangle$

$$\text{div } F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

$$= y + z + 0$$

$$= y + z$$

$$\text{curl } F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & y^2 - x^3 \end{vmatrix}$$

$$\hat{i}\left(\partial/\partial y\,(y^2 - x^3) - \partial/\partial z\,(yz) \right)$$

$$-\hat{j}\left(\partial/\partial x\,(y^2 - x^3) - \partial/\partial z\,(xy) \right)$$

$$+\hat{k}\left(\partial/\partial x\,(yz) - \partial/\partial y\,(xy) \right)$$

$$\hat{i}(2y - y) - \hat{j}(-3x^2 - 0) + \hat{k}(0 - x)$$

$$= y\hat{i} + 3x^2\hat{j} - x\hat{k}$$

$$= \langle y, 3x^2, -x \rangle$$

25. $F = \langle x - 2zx^2, z - xy, z^2x^2 \rangle$

$$\text{div } F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = \boxed{1 - 4xz - x + 2zx^2}$$

$$\text{curl } F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x - 2zx^2 & z - xy & z^2x^2 \end{vmatrix}$$

$$\hat{i}\left(\partial/\partial y\,(z^2x^2) - \partial/\partial z\,(x - xy) \right)$$
$$-\hat{j}\left(\partial/\partial x\,(z^2x^2) - \partial/\partial z\,(x - 2zx^2) \right)$$
$$+\hat{k}\left(\partial/\partial x\,(zxy) - \partial/\partial y\,(x - 2zx^2) \right)$$

$$\Rightarrow \hat{i}(0 - 1) - \hat{j}(2z^2x + 2x^2) + \hat{k}(-y - 0)$$

$$= \langle -1, -2x^2 - 2x^2, -y \rangle$$

27. $F = \langle z - y^2,\ x + z^3,\ y + x^2 \rangle$

$$\operatorname{div} F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

$$= 0 + 0 + 0 = 0$$

$$\operatorname{curl} F = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z - y^2 & x + z^3 & y + x^2 \end{vmatrix}$$

$$\hat{\imath}\left(\partial/\partial y\,(y + x^2) - \partial/\partial z\,(x + z^3)\right)$$

$$-\hat{\jmath}\left(\partial/\partial x\,(y + x^2) - \partial/\partial z\,(z - y^2)\right)$$

$$+\hat{k}\left(\partial/\partial x\,(x + z^3) - \partial/\partial y\,(z - y^2)\right)$$

$$= \hat{\imath}\,(1 - 3z^2) - \hat{\jmath}\,(2x - 1) + \hat{k}\,(1 + 2y)$$
$$= \langle 1 - 3z^2,\ 1 - 2x,\ 1 + 2y \rangle$$