15.6

13. $G(u,v) = (3u+4v, u-2v)$

$$\begin{vmatrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{vmatrix} = \begin{vmatrix} \frac{d}{du}(3u+4v) & \frac{d}{dv}(3u+4v) \\ \frac{d}{du}(u-2v) & \frac{d}{dv}(u-2v) \end{vmatrix}$$

$$= \begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} \quad \text{Jac}(G) = (3)(-2) - (4)(1) = \boxed{-10}$$

15. $G(r,t) = (r\sin t, r - \cos t)$ $(r,t) = (1, \pi)$

$$\begin{vmatrix} \frac{d}{dr}(r\sin t) & \frac{d}{dt}(r\sin t) \\ \frac{d}{dr}(r-\cos t) & \frac{d}{dt}(r-\cos t) \end{vmatrix}$$

$$\begin{vmatrix} \sin t & r\cos t \\ 1 & \sin t \end{vmatrix} \quad \text{Jac}(G) = \sin^2 t - r\cos t$$

$$(1,\pi) = \boxed{1}$$

17. $G(r,\theta) = (r\cos\theta, r\sin\theta)$ $(r,\theta) = (4, \pi/6)$

$$\begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta \ \Big|_{(4,\pi/}$$

$$= \boxed{4}$$

19. Find a linear mapping that maps $[0,1]\times[0,1]$ to parallelogram $(2,3)$ and $(4,$

$$\varphi(u,v) = (4u + 2v, u + 3v)$$

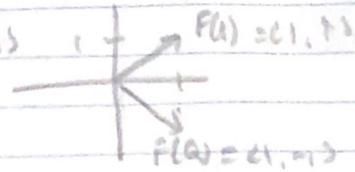
23. $G(u,v) = (3u+v, u-2v)$ $R = [0,3]\times[0,5]$ $R = [2,5]\times[1,7]$

Area $= 7 \cdot$ Area(R) Area $= 15, 18$

$$\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} = 7 \qquad = 105, 126$$

16.1

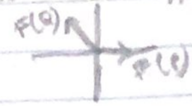
1. at $x=1$ $F(P) = \langle 1,1 \rangle$ at $x=-1$ $F(Q) = \langle 1,-1 \rangle$



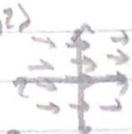
3. $P=(0,1,1)$ $Q=(2,1,0)$ $F=\langle xy, z^2, x \rangle$

$F(P) = \langle 0,1,0 \rangle$

$F(Q) = \langle 2,0,2 \rangle$



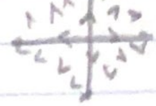
5. $F=\langle 1,0 \rangle$



7. $F = x\,i$

9. $F=\langle 0,x \rangle$

11. $F = \left\langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right\rangle$



17. $F=\langle 1,1,1 \rangle$ (c)

23. $F=\langle xy, yz, y^2-x^3 \rangle$

$\operatorname{div}(F) = y+z$ $\operatorname{curl}(F) = \langle y, 3x^2, -x \rangle$

25. $F=\langle x-2zx^2, z-xy, z^2x^2 \rangle$ $\operatorname{div}(F) = 1-4xz-x+2x^2z$

$\operatorname{curl}(F) = \langle -1, 2x^2-2xz^2, -y \rangle$

27. $F=\langle z-y^2, x+z^3, y+x^2 \rangle$ $\operatorname{div}(F)=0$

$\operatorname{curl}(F) = \langle 1-3z^2, 1-2x, 1+2y \rangle$