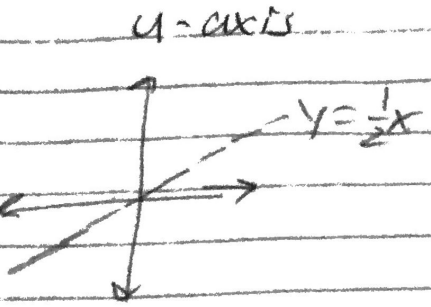
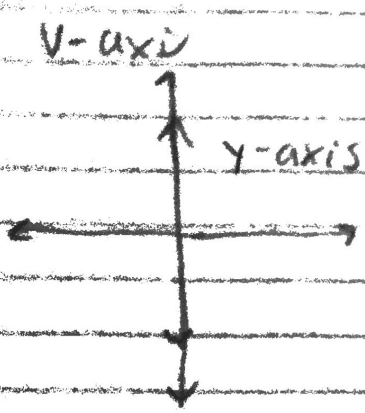
12.6, 16.1 Homework

12.6

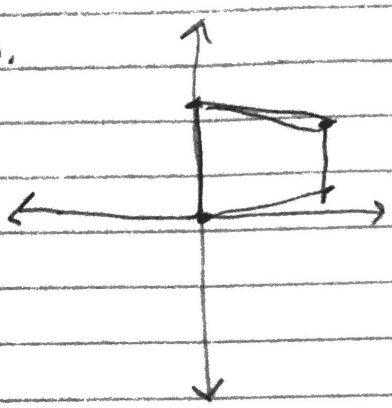
1.

a. $x = 2u$ $\quad y = u + v$

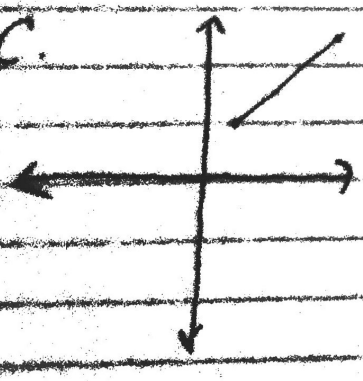




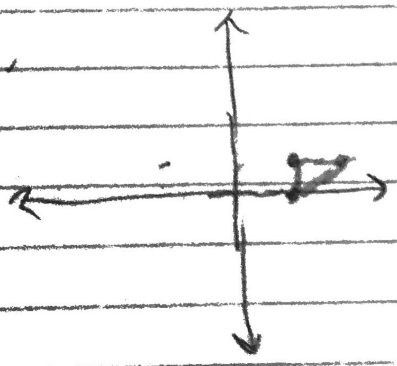
b.



c.

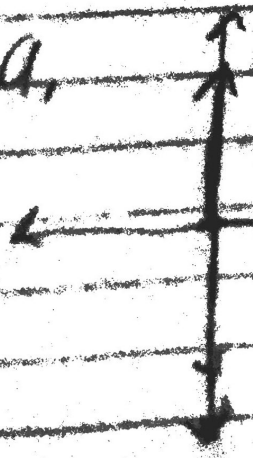


d.



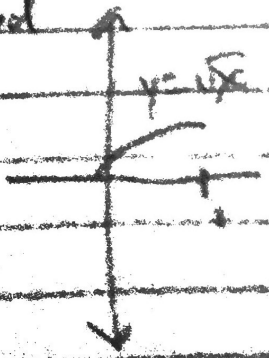
3. G is not one-one and not sure what domains it would be

a.



b.

c.



13.

$$= \begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} = -6 - 4 = \boxed{-10}$$

15.

$$= \begin{vmatrix} \sin t & r\cos t \\ 1 & \sin t \end{vmatrix} = \sin^2 t - r\cos t$$

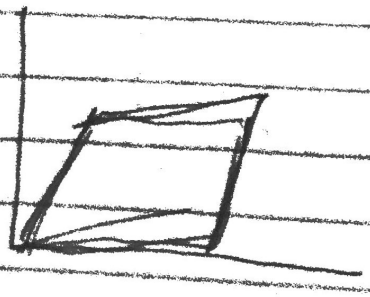
$$= 0 - \ln(1) = \boxed{1}$$

17.

$$= \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta$$

$$= r(1) = \boxed{4}$$

19. $g(u,v) = (4u+2v,\ u+3v)$



23. a.

$$\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} \quad -6 - 1 = -7$$

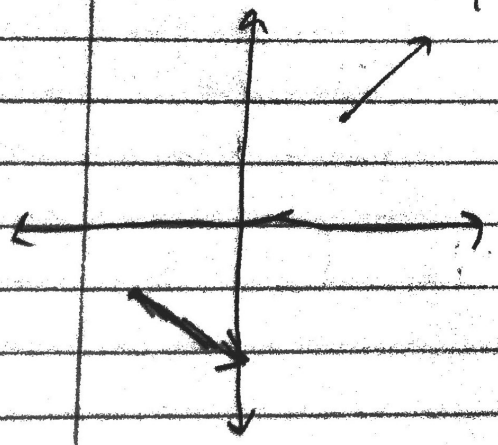
$$\iint_R (3u + v + u + 2v)\, 7 \, dA = 105$$

b $\iint (3u + v + u - 2v)\, 7 \, dA$, diff bounds

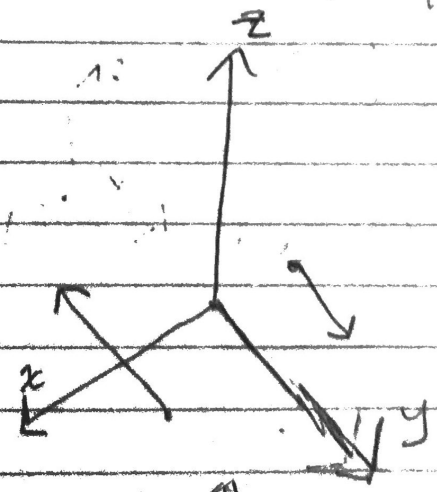
$= -126$

16.1

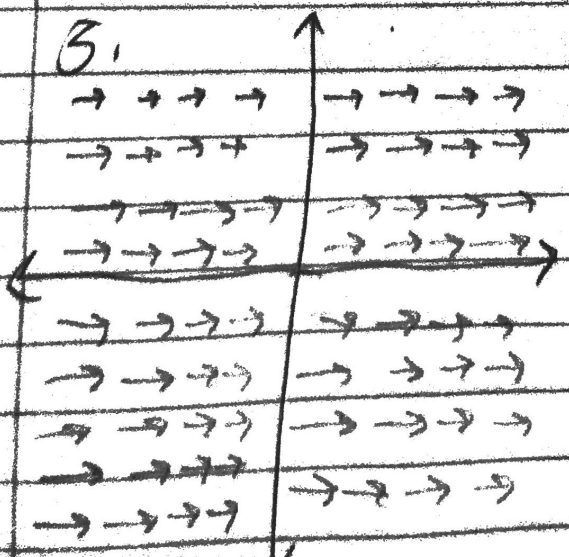
1. $F = \langle 1, 1 \rangle$ $\langle 1, -1 \rangle$



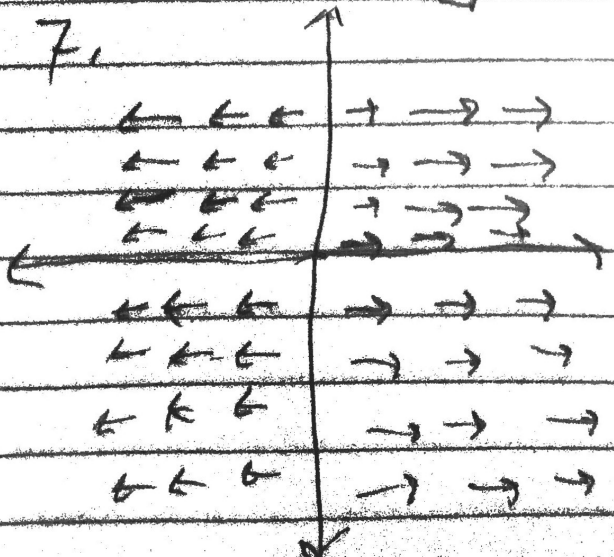
3. $F = \langle 0, 1, 0 \rangle$ $\langle 2, 0, 2 \rangle$



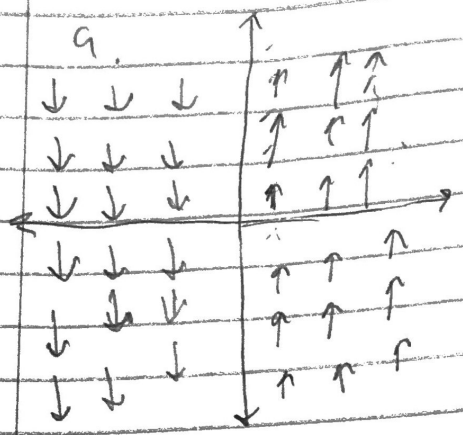
5.



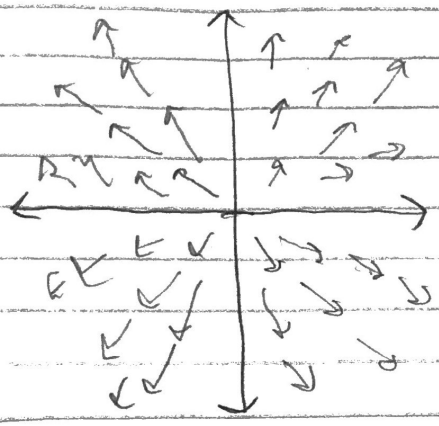
7.



9.



11.



17. C

23. $\nabla \times F$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & y^2 - x^3 \end{vmatrix}$$

$$= i\left(\frac{d}{dy}(y^2 - x^3) - \frac{d}{dz}(yz)\right) + j\left(\frac{d}{dx}(y^2 - x^3) - \frac{d}{dz}(xy)\right) + k\left(\frac{d}{dx}(yz) - \frac{d}{dy}(xy)\right)$$

$$= (2y - y) - (3x^2 - 0)j + (0 - x)$$

$$= iy - j3x^2 + kx$$

$$\text{Curl}(F) = \langle y, 3x^2, -x \rangle$$

$$\text{div}(F) = y + z$$

25. $\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x-2zy^2 & z+xy & z^3x^2 \end{vmatrix}$

$\text{Curl}(F) = i\left(\frac{\partial}{\partial y}(z^3x^2) - \frac{\partial}{\partial z}(z+xy)\right) - j\left(\frac{\partial}{\partial x}(z^3x^2) - \frac{\partial}{\partial z}(x-2zy^2)\right)$
$+ k\left(\frac{\partial}{\partial x}(z+xy) - \frac{\partial}{\partial y}(x-2zy^2)\right)$

$= i(0-1) - j(2xz^3 - 2x^2) + k(y-0)$

$\boxed{\text{Curl}(F) = \langle -1, 2xz^3 - 2x^2, y \rangle}$
$\boxed{\text{div}(F) = 1 - 4xz - y - 2zy^2}$

27. $\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z-y^2 & x+z^3 & y+x^2 \end{vmatrix}$

$\text{Curl}(F) = i\left(\frac{\partial}{\partial y}(y+x^2) - \frac{\partial}{\partial z}(x+z^3)\right) - j\left(\frac{\partial}{\partial x}(y+x^2) - \frac{\partial}{\partial z}(z-y^2)\right)$
$+ k\left(\frac{\partial}{\partial x}(x+z^3) - \frac{\partial}{\partial y}(z-y^2)\right)$

$= i(1-3z^2) - j(2x-y^2) + k(1-2y)$

$\boxed{\text{Curl}(F) = \langle 1-3z^2, y^2-2x, 1-2y \rangle}$
$\boxed{\text{div}(F) = 0}$