16.4.

7.

$r=(2u+v)i+(u-4v)j+3uk$

$r_u=2i+j+3k$

$r_v=i-4j+0k$

$u=1$ $v=4$

$r_{u(1,4)}=2i+j+3k$

$r_{v(1,4)}=i-4j+0$

$r_u=\langle 2,1,3\rangle$ $r_v\langle 1,-4,0\rangle$

$$\begin{matrix} i & j & k \\ 2 & 1 & 3 \\ 1 & -4 & 0 \end{matrix}$$

$=0+12i-(0-3j)-9k$

$N\langle 12,3,-9\rangle$

$\langle 6,-5,3\rangle$

$12(x-6)+3(y+5)-9(z-3)=0$

$12x-72+3y+45-9z+27=0$

$12x+3y-9z=0$

$4x+y-3z=0$

15. $r=\langle x,9-z^2,z\rangle$

$r_x=\langle 1,0,0\rangle$

$r_z=\langle 0,-2z,1\rangle$

$$\begin{matrix} i & j & k \\ 1 & 0 & 0 \\ 0 & -2z & 1 \end{matrix}$$

$2zi-j+0-2zk$

$=\langle 2z,-1,-2z\rangle$

$|r_x\times r_z|=\sqrt{4z^2+1+4z^2}=\sqrt{8z^2+1}$

$\iint_S f(x,y,z)\,dS=\int_0^3\int_0^3 z\cdot\sqrt{8z^2+1}\,dz\,dx$

$=77.839.$

13.

$f(G(u,v))=\cancel{u(u\cos v)^2+}\;u(u\cos v)^2+(u\sin v)^2$

$=u^3\cos^2 v+u^3\sin^2 v$

$=u^3(\cos^2 v+\sin^2 v)$

$=u^3\times 1$

$=u^3$

$G_u=\langle\cos v,\sin v,1\rangle$

$G_v=\langle -u\sin v,u\cos v,0\rangle$

$G_u\times G_v=\langle -u\cos v,-u\sin v,u\rangle.$

$|G_u\times G_v|=\sqrt{u^2\cos^2 v+u^2\sin^2 v+u^2}=u\sqrt{2}$

$\iint_S f(x,y,z)\,dS$

$=\int_0\int_0^1 u^3\cdot u\sqrt{2}\,du\,dv$

$=\frac{\sqrt{2}}{5}$

19.

$x^2+y^2=4$

$r=2$.

$x=2\cos u$

$y=2\sin u$

$z=v$.

$0\le u\le 2\pi$

$0\le v\le 4$.

$r(u,v)=\langle 2\cos u, 2\sin u, v\rangle$

$r_u=\langle -2\sin u, 2\cos u, 0\rangle$

$r_v=\langle 0, 0, 1\rangle$

$r_u\times r_v$

$$=\begin{vmatrix} i & j & k \\ -2\sin u & 2\cos u & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$2\cos u\, i+2\sin u\, j+0$

$=\langle 2\cos u, 2\sin u, 0\rangle$

$|r_u\times r_v|=\sqrt{4\cos^2u+4\sin^2u+0}$

$=2$.

$\iint_S f(x,y,z)\,dS$

$=\int_0^4\int_0^{2\pi} e^{-v}\cdot 2\,du\,dv$

$=4\pi(1-e^{-4})$

16.5

5. $g = 1-3x+4y$

$P = y \quad Q = z \quad R = x$

$\iint_D -y(-3) - z(4) + x \, dA$

$\iint_D 3y - 4z + x \, dA$

$z = 1-3x+4y$

$\iint_D 3y - 4(1-3x+4y) + x \, dA$

$\iint_D 3y - 4 + 12x - 16y + x \, dA$

$0 \le x \le 1 \quad 0 \le y \le 1$

$\int_0^1 \int_0^1 (3y - 4 + 12x - 16y + x) \, dx dy$

$\int_0^1 \int_0^1 (-13y + 13x - 4) \, dx dy$

$= -4$

7. $\cancel{g = \sqrt{9-x^2-y^2}}$

$\cancel{P = 0} \quad Q = 3 \quad R = x$

$\cancel{\nabla F = f_x x 0 + f_y x 3 + f_z} x$

$P = 0 \quad Q = 3 \quad R = x$

$\cancel{\nabla F} = 0$

$g = \sqrt{9-x^2-y^2}$

$\iint_S \quad \frac{dg}{dx} = \frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}} \cdot (-2x)$

$\frac{dg}{dy} = \frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}} \cdot (-2y)$

$\iint_D 0 \cdot -3(\frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}} \cdot (-2y)) + x \, dA$

$\iint_D \frac{3y}{\sqrt{9-x^2-y^2}} + x \, dA$

$\int_0^3 \int_0^{\sqrt{9-x^2-y^2}} \frac{3y}{\sqrt{9-x^2-y^2}} + x \, dx dy$

$= 30.2058$

9. $g = 9-x^2-y^2$

$P = z \quad Q = z \quad R = x$

$\frac{dg}{dx} = -2x \quad \frac{dg}{dy} = -2y$

$\iint_D -z(-2x) - z(-2y) + x \, dA$

$= \iint_D 2zx + 2yz + x \, dA$

$= \iint_D 2z(x+y) + x \, dA$

$= \int \int_0^{\sqrt{9-x^2}} 2 \cdot (9-x^2-y^2)(x+y) + x \, dx dy$

$= \frac{693}{5}$

11. $F = \langle y^2, 2, -x \rangle$

$g = 1-x-y$

$P = y^2 \quad Q = 2 \quad R = -x$

$\frac{dg}{dx} = -1 \quad \frac{dg}{dy} = -1$

$\iint_D \cancel{y^2} \, y^2 + 2 - x \, dA$

$\int_0^1 \int_0^{1-y} y^2 + 2 - x \, dx dy$

$= \frac{4}{12}$