Math 251 Shaun Goda Section 23 HW #11

January February March April May June July August September October November December
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

16.4:

7) $T_u = \langle 2, 1, 3 \rangle$
$T_v = \langle 1, -4, 0 \rangle$

$T_u \times T_v = \langle 2, 1, 3 \rangle \times \langle 1, -4, 0 \rangle = \begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ 1 & -4 & 0 \end{vmatrix} = (+12)i - (-3)j + (-8-1)k$
$= \langle 12, 3, -9 \rangle$

$N(x, y) = 4x + y - 3z = 0$

13) $\iint_S z(x^2 + y^2)\, dS = \iint_D u((u\cos v)^2 + (u\sin v)^2) |\langle \cos v, \sin v, 1 \rangle \times \langle -u\sin u, u\cos v, 0 \rangle| dA$

$r_u \times r_v = \langle \cos v, \sin v, 1 \rangle \times \langle -u\sin v, u\cos v, 0 \rangle$

$= \begin{vmatrix} i & j & k \\ \cos v & \sin v & 1 \\ -u\sin v & u\cos v & 0 \end{vmatrix} = (u\cos^2 v + u\sin^2 v)i - (u\sin v)j + (u\cos^2 v + u\sin^2 v)k$

$= \langle u, -u\sin v, u \rangle$

$u(u^2\cos^2 v + u^2\sin^2 v) = u^3$
$u^3 \langle u, -u\sin v, u \rangle = u^4 - u^4\sin v + u^4$

$\iint_S z(x^2 + y^2)\, dS = \int_0^1 \int_0^1 (u^4 - u^4 \sin v + u^4)\, du\, dv$

$= \frac{\cos(1)}{5} + \frac{1}{5} \approx 0.308$

15) $\iint_S z\, dS = \iint_D \sqrt{9-y}\cdot\sqrt{0^2 + \left(\frac{1}{2\sqrt{9-y}}\right)^2 + 1}\, dA$

$= \int_0^3\int_0^3 \sqrt{9-y}\cdot\sqrt{\frac{(2\sqrt{9-y}-1)\sqrt{9-y}}{2(-y+9)}}\, dy\, dx$

$\approx$ using calc, 25.02

19) $\iint_S e^{-z} dS = \iint_D e^{-4-x^2-y^2}\cdot\sqrt{(-2x)^2+(-2y)^2+1}\, dA$

$= \int_0^{\sqrt{2}}\int_0^{\sqrt{2}} e^{-4-x^2-y^2}\sqrt{(-2x)^2+(-2y)^2+1}\, dy\, dx = 0.025$

$x^2+y^2+0 = 4$
$x = \sqrt{2} \quad y = \sqrt{2}$
$x^2+y^2+4 = 4$
$x = 0 \quad y = 0$

16.5:

5) $\iint_S \langle y, z, x\rangle dS = \iint_R \langle y, \overset{1+4y-3x}{\cancel{z}}, x\rangle \cdot \langle 3, -4, 1\rangle\, dA$

$= \iint_R (3y - 4 - 16y + 12x + x)\, dA$

$= \iint_R (-13y + 13x - 4)\, dA$

$= \int_0^1\int_0^1 (-13y + 13x - 4)\, dy\, dx = -4$

$$z = \sqrt{9 - x^2 - y^2}$$

7) $$\iint_S \langle 0, 3, x \rangle \, dS = \iint_R \langle 0, 3, x \rangle \left\langle \frac{x}{\sqrt{-x^2 - y^2 + 9}}, \frac{y}{\sqrt{-y^2 - x^2 + 9}}, 1 \right\rangle dA$$

$$= \int_0^1 \int_0^1 \left(0 + \frac{3y}{\sqrt{-x^2 - y^2 + 9}} + x \right) dy\, dx$$

$$= 1.0254$$

9) $$\iint_S \langle z, z, x \rangle \, dS = \iint_R \langle 9 - x^2 - y^2, 9 - x^2 - y^2, x \rangle \langle 2x, 2y, 1 \rangle \, dA$$

$$= \int_0^1 \int_0^1 \left(18x - 2x^3 - 2xy^2 + 18y - 2x^2 y - 2y^3 + x \right) dy\, dx$$

$$= \frac{103}{6} - 2(0)(0)^2 = 17.166$$

11) $$\iint_S \langle xz, yz, z^{-1} \rangle = \iint_R \left\langle x\sqrt{\tfrac{27}{4} - x^2 - y^2}, y\sqrt{\tfrac{27}{4} - x^2 - y^2}, \frac{1}{\sqrt{\tfrac{27}{4} - x^2 - y^2}} \right\rangle \cdot \left\langle \frac{-2x}{\sqrt{-4x^2 - 4y^2 + 27}}, \frac{-2y}{\sqrt{-4x^2 - 4y^2 + 27}}, 1 \right\rangle$$

$$\frac{4z^2}{3} + x^2 + y^2 = 9$$

$$z = \sqrt{\frac{27}{4} - x^2 - y^2}$$

$$= \int_0^1 \int_0^1 \left(\frac{-2x^2\sqrt{\tfrac{27}{4} - x^2 - y^2}}{\sqrt{-4x^2 - 4y^2 + 27}} - \frac{2y^2\sqrt{\tfrac{27}{4} - x^2 - y^2}}{\sqrt{-4x^2 - 4y^2 + 27}} + \frac{1}{\sqrt{\tfrac{27}{4} - x^2 - y^2}} \right) dA \to dy\,dx$$

$$\approx 0.26$$