16.4 & 16.5 (Nov. 22nd)

16.4: # 7, 13, 15, 19

16.5: # 5, 7, 9, 11

16.4: # 7, 13, 15, 19

7) $G(u,v) = (2u+v, u-4v, 3u)$; $u=1, v=4$

$r_u = 2\hat{i} + \hat{j} + 3\hat{k}$

$r_v = 1\hat{i} - 4\hat{j} + 0\hat{k}$

at 1, 4

$T_u = \langle 2, 1, 3 \rangle$

$T_v = \langle 1, -4, 0 \rangle$

$$N = T_u \times T_v = \begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ 1 & -4 & 0 \end{vmatrix}$$

$n = (0+12)\hat{i} - (0-3)\hat{j} + (-8-1)\hat{k}$

$= 12\hat{i} + 3\hat{j} - 9\hat{k}$

plug in $u=1, v=2$ into original

$(6, -15, 3)$

$12(x-6) + 3(y+15) - 9(z-3) = 0$

$12x - 72 + 3y + 45 - 9z + 27 =$

$4x + y - 3z = 0$

13) $G(u,v) = (u\cos v, u\sin v, u)$ $\quad 0 \le r \le 1, \ 0 \le \theta \le 1$

$G_u = \langle \cos v, \sin v, 1 \rangle$

$G_v = \langle -u\sin v, u\cos v, 0 \rangle$

$|G_u \times G_v| = \sqrt{2u^2} = u\sqrt{2}$

$\iint F\,dS = \sqrt{2}\int_0^1 \int_0^1 u^4\,du\,dv = \sqrt{2}\int_0^1 \left[\frac{1}{5}u^5\right]_0^1 dv$

$= \sqrt{2}\int_0^1 \frac{1}{5}\,dv = \boxed{\frac{\sqrt{2}}{5}}$

15) $y = 9 - z^2$ $0 \le x \le 3$ $f(x, y, z) = z$

$y + 9 = -z^2$ $0 \le z \le 3$

$9 - y = z^2$

$\sqrt{9-y} = z$

$\int_0^3 \int_0^3 z\sqrt{0 + (-2z)^2 + 1}\, dz\, dx$

$(9-y)^{1/2}$

$\frac{1}{2}(9-y)^{-1/2}$

$= \int_0^3 z\sqrt{1+4z^2}\, dz$

$-\frac{1}{2\sqrt{9-y}}$

$u = 1 + 4z^2$

$z\,dz = \frac{du}{8}$

$2\int_1^{37} \sqrt{u}\, \frac{du}{8}$

$= 2[u^{3/2}]_1^{37}$

$= \boxed{\frac{37\sqrt{37} - 1}{4}}$

19) $x = 2\cos(u)$, $y = 2\sin(u)$

$0 \le u \le 2\pi$

$z = v$, so $0 \le v \le 4$

$\vec{r}(u, v) = \langle 2\cos u, 2\sin u, v \rangle$

$\iint f(x, y, z)\, dS = \iint_D f(\vec{r}(u, v))\, |\vec{r}_u \times \vec{r}_v|\, dA$

$f(\vec{r}(u, v)) = e^{-v}$

$\vec{r}_u = \langle -2\sin u, 2\cos u, 0 \rangle$

$\vec{r}_v = \langle 0, 0, 1 \rangle$

$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} i & j & k \\ -2\sin u & 2\cos u & 0 \\ 0 & 0 & 1 \end{vmatrix} = 2\cos u\,\hat{i} + 2\sin u\,\hat{j} + 0\,\hat{k}$

$|\vec{r}_u \times \vec{r}_v| = \sqrt{2\cos(u)^2 + 2\sin(u)^2 + 0^2} = 2$

$\int_0^{2\pi} \int_0^4 2e^{-v}\, dv\, du$

$= 2\int_0^{2\pi} [-e^{-4} + e^{-0}]\, du$

$= \boxed{4\pi[1 - e^{-4}]}$

16.5: # 5, 7, 9, 11

5) $z = 1-3x+4y$

$\phi(x,y) = (x, y, 1-3x+4y)$

$D = \{(x,y) : 0 \leq x, y \leq 1\}$ in the xy plane.

$\vec{T_x} = \frac{\partial \phi}{\partial x} = \langle 1, 0, -3 \rangle$

$\vec{T_y} = \frac{\partial \phi}{\partial y} = \langle 0, 1, 4 \rangle$

$T_x \times T_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -3 \\ 0 & 1 & 4 \end{vmatrix} = \langle 3, -4, 1 \rangle$

$F(\phi(x,y)) = \langle y, z, x \rangle = \langle y, 1-3x+4y, x \rangle \cdot n$

$= 13x - 13y - 4$

$\iint_S F\,ds = \int_0^1\int_0^1 13x - 13y - 4 \; dx\,dy$

$= \int_0^1 \frac{13x^2}{2} - 13xy - 4x \Big|_0^1 = \frac{5y}{2} - \frac{13y^2}{2}\Big|_0^1 = \boxed{-4}$

7) $\phi(\theta, \phi) = (3\cos\theta \sin\phi, 3\sin\theta\sin\phi, 3\cos\phi)$

$0 \leq \theta \leq \frac{\pi}{2}$, $0 \leq \phi \leq \frac{\pi}{2}$

$n = T_\phi \times T_\theta = 9\sin\phi \langle \cos\theta\sin\phi, \sin\theta\sin\phi, \cos\phi \rangle$

$0 \leq \theta \leq \frac{\pi}{2}$, $0 \leq \phi \leq \frac{\pi}{2}$, n is positive

$F(\phi(\theta,\phi)) \cdot n(\theta,\phi) = \langle 0, 3, 3\cos\theta\sin\phi \rangle \cdot 9\sin\phi \langle \cos\theta\sin\phi, \sin\theta\sin\phi, \cos\phi \rangle$

$= 27\sin\theta\sin^2\phi + 27\cos\theta\sin^2\phi\cos\phi$

$\int_0^{\pi/2}\int_0^{\pi/2} 27\sin\theta\sin^2\phi + 27\cos\theta\sin^2\phi\cos\phi \; d\theta\, d\phi$

$= 27\left(-\cos\theta\Big|_0^{\pi/2} - \left(\frac{\phi}{2} - \frac{\sin 2\phi}{4}\right)\Big|_0^{\pi/2} + \sin\theta\Big|_0^{\pi/2} - \frac{\sin^3\phi}{3}\Big|_0^{\pi/2}\right)$

$= \boxed{\frac{27}{12}(3\pi+4)}$

9) $z_x = -2x$, $z_y = -2y$

$N = \langle 2x, 2y, 1 \rangle$, $f \cdot N = \langle 2, 2, x \rangle \cdot \langle 2x, 2y, 1 \rangle$

$= (2x + 2y)(2) + x$

$x^2 + y^2 = 9$

$y = \sqrt{9 - x^2}$ $\quad 0 \le y \le \sqrt{9 - x^2}$

$0 \le x \le 3$

$\int_0^3 \int_0^{\sqrt{9-x^2}} (2)(x+y)(9 - x^2 - y^2) + x \, dy \, dx$

$x = r\cos\theta$, $y = r\sin\theta \rightarrow x^2 + y^2 = r^2 = 9 \rightarrow 0 \le r \le 3$

$0 \le \theta \le \frac{\pi}{2}$

$= \int_0^{\pi/2} \int_0^3 (2(8)(\cos\theta + \sin\theta)(9 - r^2) + r\cos\theta) \, r \, dr \, d\theta = \boxed{\frac{693}{5}}$

11) $f = \langle y^2, 2, -x \rangle$

$x + y + z = 1 \rightarrow z = 1 - x - y$

$\iint_S f \, ds = -\iint_D (-Pf_x - Qf_y + R) \, dA$

$= -\iint_D (y^2 + 2 - x) \, dy \, dx \leftarrow -\int_D \int (y^2)(-1) - (2)(1) - x \, dA$

$\int_0^1 \int_0^{1-x} (x - y^2 - 2) \, dy \, dx$

$= \frac{x^3}{3} - 2x^2 + 4x - \frac{7}{3}$

$\int_0^1 \frac{x^3}{3} - 2x^2 + 4x - \frac{7}{3}$

$= \frac{1}{12} - \frac{2}{3} + 2 - \frac{7}{3} = \boxed{\frac{11}{12}}$