HW Due 11/22

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16.4 → # 7, 13, 15, 19:

7. $G(u,v) = (2u+v,\ u-4v,\ 3u)$; $u=1$, $v=4$

$T_u = \langle 2, 1, 3\rangle$
$T_v = \langle 1, -4, 0\rangle$

Normal vector: $T_u \times T_v = \begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ 1 & -4 & 0 \end{vmatrix}$

$n = i(12) - j(-3) + k(-9)$
$= 12i + 3j - 9k$
$= \langle 12, 3, -9\rangle$

Formula: $\langle x-x_0, y-y_0, z-z_0\rangle \cdot$ normal vector

$G(1,4) = \langle 6, -15, 3\rangle$

$\langle x-6, y+15, z-3\rangle \cdot \langle 12, 3, -9\rangle = 0$
$12(x-6) + 3(y+15) - 9(z-3) = 0$
$12x - \cancel{72} + 3y \cancel{+45} - 9z \cancel{+27} = 0$
$\boxed{12x + 3y - 9z = 0}$

13. $\iint_S f(x,y,z)\,dS$

$G(u,v) = (u\cos v, u\sin v, u)$, $0 \le u \le 1$, $0 \le v \le 1$;
$F(x,y,z) = z(x^2+y^2)$
$G_u = \langle \cos v, \sin v, 1\rangle$ $\quad G_v = \langle -u\sin v, u\cos v, 0\rangle$
$|G_u \times G_v| = \begin{vmatrix} i & j & k \\ \cos v & \sin v & 1 \\ -u\sin v & u\cos v & 0 \end{vmatrix}$ $\quad i(-u\cos v) - j(-u\sin v) + k(u\cos^2 v - u\sin^2 v)$

$|G_u \times G_v| = \sqrt{(-u\cos v)^2 + (u\sin v)^2 + (u\cos^2 v - u\sin^2 v)^2} = \sqrt{2u^2} = u\sqrt{2}$

$\sqrt{2}\int_0^1\int_0^1 u^4\,du\,dv \rightarrow \frac{u^5}{5}\Big]_0^1 = \sqrt{2}\int_0^1 \frac{1}{5}\,dv = \boxed{\frac{\sqrt{2}}{5}}$

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(15) $y = 9 - z^2,\ 0 \le x \le 3,\ 0 \le z \le 3;\ f(x,y,z) = z$

Let $y = 9 - z^2$ be parameterized by $G(x,z)$

$y = g(x,z)$

$G(x,z) = (x, 9-z^2, z)$

$T_x = \langle 1, 0, 0 \rangle$

$T_z = \langle 0, -2z, 1 \rangle$

$$T_x \times T_z = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & -2z & 1 \end{vmatrix}$$

$i(0) - j(1) + k(-2z)$

$= -j - 2zk$

$n = \langle 0, -1, -2z \rangle$

$\|n\| = \sqrt{(-1)^2 + (-2z)^2} = \sqrt{1 + 4z^2}$

$f(G(x,z)) = f(x, 9-z^2, z) = z \Rightarrow$ in problem instructions

$$\int_0^3 \int_0^3 z \cdot \sqrt{1+4z^2}\, dz\, dx$$

Outer Integral = 3

Inner:

$u = 1 + 4z^2$

$\frac{du}{8} = \frac{8z}{8}$

$$\frac{1}{8} \int_0^3 \sqrt{u}\, du$$

$$\frac{2u^{3/2}}{3} \Big]_0^3 = \frac{2(1+4z^2)^{3/2}}{3} \Big]_0^3$$

$$\frac{3}{8}\left(\frac{\sqrt{37}^3 (2)}{3} - \frac{2}{3} \right)$$

$= \boxed{56.02}$

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16.4 → #19:

19 $x^2+y^2=4$, $0 \le z \le 4$, $F(x,y,z)=e^{-z}$

In cylindrical coordinates:

$$\phi(\theta,z) = (2\cos\theta, 2\sin\theta, z), \quad 0 \le \theta \le 2\pi,$$
and $0 \le z \le 4$

$$T_\theta = \langle -2\sin\theta, 2\cos\theta, 0 \rangle$$
$$T_z = \langle 0, 0, 1 \rangle$$

$$T_\theta \times T_z = \begin{vmatrix} i & j & k \\ -2\sin\theta & 2\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = 2\cos\theta\, i + 2\sin\theta\, j$$

$$\| T_\theta \times T_z \| = \sqrt{4\cos^2\theta + 4\sin^2\theta} = \sqrt{4} = 2$$

$$\int_0^{2\pi}\int_0^4 e^{-z}\, 2\, dz\, d\theta$$

Outer:

$$\int_0^{2\pi} 2\, d\theta = 4\pi$$

Inner:

$$\int_0^4 e^{-z}\, dz = -e^{-z}\Big]_0^4 = (-e^{-4}+1)4\pi$$

$$= \boxed{4\pi(1-e^{-4})}$$

16.5-D # 5, 7, 9, 11;

(5) compute $\iint_S F \cdot dS$

$F = \langle y, z, x \rangle$ plane $3x - 4y + z = 1$ $0 \le x \le 1, 0 \le y \le 1$, $\underbrace{\text{upward}}_{xy \text{ plane}}$ pointing normal)

$$z = 1 - 3x + 4y$$

$$\Phi(x,y) = \langle x, y, 1 - 3x + 4y \rangle$$

$$T_x = \langle 1, 0, -3 \rangle$$

$$T_y = \langle 0, 1, 4 \rangle$$

$$T_x \times T_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -3 \\ 0 & 1 & 4 \end{vmatrix} = +3i - 4j + k = \langle 3, -4, 1 \rangle$$

$$F(\Phi(x,y)) \cdot n = \langle y, 1 - 3x + 4y, x \rangle \cdot \langle 3, -4, 1 \rangle$$

$$= 13x - 13y - 4$$

$$\int_0^1 \int_0^1 (13x - 13y - 4)\, dx\, dy$$

Inner: $\left. \frac{13x^2}{2} - 13xy - 4x \right]_0^1 = \int_0^1 \left(\frac{13}{2} - 13y - \frac{4}{1} \right) dy$

$$= \left. \frac{5y}{2} - \frac{13y^2}{2} \right]_0^1$$

$$= \frac{5}{2} - \frac{13}{2} = -\frac{8}{2} = \boxed{-4}$$

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16.5 → #7, 9, 11:

7 $F = \langle 0, 3, x \rangle$ part of sphere $x^2+y^2+z^2=9$
where $x \geq 0, y \geq 0, z \geq 0$, outward pointing normal

Spherical coordinates:

$$\phi(\theta,\phi) = (3\cos\theta\sin\phi,\ 3\sin\phi\sin\theta,\ 3\cos\phi)$$
$$0 \leq \theta \leq \pi/2,\ 0 \leq \phi \leq \frac{\pi}{2}$$

$$T_\theta = \langle -3\sin\theta\sin\phi,\ 3\sin\phi\cos\theta,\ 0 \rangle$$
$$T_\phi = \langle 3\cos\theta\cos\phi,\ 3\cos\phi\sin\theta,\ -3\sin\phi \rangle$$

$$T_\theta \times T_\phi = \begin{vmatrix} i & j & k \\ -3\sin\theta\sin\phi & 3\sin\phi\cos\theta & 0 \\ 3\cos\theta\cos\phi & 3\cos\phi\sin\theta & -3\sin\phi \end{vmatrix} = 9\sin\phi\langle\cos\theta\sin\phi, \sin\theta\sin\phi, \cos\phi\rangle$$

for $0 \leq \phi \leq \pi/2,\ 0 \leq \theta \leq \pi/2$

$$F(\phi(\theta,\phi)) = \langle 0, 3, x\rangle = \langle 0, 3, 3\cos\theta\sin\phi\rangle$$
$$F(\phi(\theta,\phi))\cdot n = \langle 0,3,3\cos\theta\sin\phi\rangle \cdot 9\sin\phi\langle\cos\theta\sin\phi, \sin\theta\sin\phi, \cos\phi\rangle$$
$$= \int_0^{\pi/2}\int_0^{\pi/2} 27\sin\theta\sin^2\phi + 27\cos\theta\sin^2\phi\cos\phi\, d\theta\, d\phi$$

Done on maple

$$= \frac{27}{12}(3\pi+4)$$

9 $$\phi(x,y) = (x, y, 9-x^2-y^2)$$
$$T_x = \langle 1, 0, -2x\rangle$$
$$T_y = \langle 0, 1, -2y\rangle$$
$$T_x \times T_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -2x \\ 0 & 1 & -2y \end{vmatrix} = 2xi + 2yj + k$$
$$= \langle 2x, 2y, 1\rangle$$

$$F(\phi(x,y))\cdot n = \langle 9-x^2-y^2, 9-x^2-y^2, x\rangle \cdot \langle 2x, 2y, 1\rangle$$
$$= 18x - 2x^3 - 2xy^2 + 18y - 2x^2y - 2y^3 + x$$
$$= 19x + 18y - 2xy(x^2+y^2)$$

$$\iint_S F \cdot dS = \iint_D F(\Phi(x,y)) \cdot n(x,y)\,dx\,dy$$

Polar coordinates $= \int_0^3 \int_0^{\pi/2} (19r\cos\theta + 18r\sin\theta - 2r^2\sin\theta\cos\theta)\,d\theta\,dr$

$0 \le y < 3$

$0 \le x \le \pi/2$

$\Downarrow$ Done On Maple

$= \frac{693}{5} = \boxed{138.6}$

11 $F = y^2 i + 2j - xk$ portion of plane $x+y+z=1$ in octant $x,y,z \ge 0$ upward pointing

$\Phi(x,y) = \langle y^2, 2, -x \rangle$

$T_x = \langle 0, 0, -1 \rangle$

$T_y = \langle 2y, 0, 0 \rangle$

$$T_x \times T_y = \begin{vmatrix} i & j & k \\ 0 & 0 & -1 \\ 2y & 0 & 0 \end{vmatrix}$$

$i(0) - j(+2y) + k(0)$

$= -2y\,j$

$= \langle 0, -2y, 0 \rangle$

$\langle x, y, 1-x-y \rangle \cdot \langle 0, -2y, 0 \rangle$

$= -2y^2$

$$\int_0^1 \int_0^{\pi/2} -2r\sin^2\theta\,d\theta\,dr$$

$= \frac{11}{12} \rightarrow$ Done On Maple