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16.4: 7, 13, 15, 17

16.5: 5, 7, 9, 11

①  $G(u, v) = (2u + v, u - 4v, 3u)$

$u = 1 \quad T_u = \langle 2, 1, 3 \rangle$   
 $v = 4 \quad T_v = \langle 1, -4, 0 \rangle$

$T_u \times T_v = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -4 & 0 \end{vmatrix} = 4i + 3j - 11k$

$= \langle 4, 3, -11 \rangle = 3 \langle 4, 1, -3 \rangle$

Point =  $(4, -15, 3) = (2, -5, 1)$

Eq:  $4(x-2) + (y+5) - 3(z-1) = 0$   
 $4x - 8 + y + 5 - 3z + 3 = 0$   
 $4x + y - 3z = 0$

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$G(u, v) = (u \cos v, u \sin v, u)$   
 $f(x, y, z) = z(x^2 + y^2)$

$0 \leq u \leq 1$   
 $0 \leq v \leq 1$   
 $g_u = 1^2$   
 $g_v = 0$

$\iint_R (z(x^2 + y^2)) \sqrt{1 + 0 + 1} \, dA$

$= \int_0^1 \int_0^1 u \cdot u^2 \sqrt{2} \, dv \, du = \sqrt{2} \int_0^1 u^3 \, du = \sqrt{2} \left[ \frac{u^4}{4} \right]_0^1 = \frac{\sqrt{2}}{4}$

$\sqrt{2} \int_0^1 u^4 \, du = \frac{\sqrt{2}}{5}$

⑮  $y = 9 - z^2 \quad 0 \leq x \leq 3 \quad 0 \leq z \leq 3 \quad f(x, y, z) = z$

$\int_0^3 \int_0^3 z \sqrt{4z^2 + 1} \, dz \, dx$

$y_x = 0$   
 $y_z = -2z$   
 $\sqrt{4z^2 + 1}$

$= \int_0^3 z \sqrt{4z^2 + 1} \, dz = \left[ \frac{(4z^2 + 1)^{3/2}}{12} \right]_0^3 = 18.67$

$\int_0^3 18.67 \, dx = 56.01 = \frac{37\sqrt{37} - 1}{4}$

⑯  $x^2 + y^2 = 4 \quad 0 \leq z \leq 4 \quad f(x, y, z) = e^{-z}$

$z = 2 - r$   
 $x = r \cos \theta$   
 $y = r \sin \theta$   
 $\int_0^{2\pi} \int_0^4 e^{-r} \sqrt{4r^2 \cos^2 \theta + 4r^2 \sin^2 \theta} \, dr \, d\theta$

⑤  $F = \langle 4y, z, x \rangle \quad 11, 22, 20$

plane  $3x - 4y + z = 1$

$0 \leq x \leq 1 \quad 0 \leq y \leq 1$

$z = 1 - 3x + 4y$

$g_x = -3$

$g_y = 4$

$\int_0^1 \int_0^1 (-y(-3) - z(4) + x) \, dy \, dx$

$= \int_0^1 3y - 4z + x \, dx$

$= \int_0^1 3y - 4 + 12x - 16y + x \, dx$

$= \int_0^1 -13y + 3x - 4 \, dx$

$= \left[ -13yx + \frac{3x^2}{2} - 4x \right]_0^1$

$= -13y + \frac{3}{2} - 4$

$= -\frac{13}{2} + \frac{3}{2} - 4 = -4$

⑦  $F = \langle 0, 3, x \rangle \quad x^2 + y^2 + z^2 = 9$   
 $z = \sqrt{9 - x^2 - y^2}$

$g_x = -1$

$g_y = -1$

$\iint$

⑩  $F = \langle z, z, x \rangle \quad z = 9 - x^2 - y^2$

$x \geq 0, y \geq 0, z \geq 0$

$\int_0^1 \int_0^1 2xz + 2yz + x \, dy \, dx$

$= \int_0^1 19x - 2xz - 2xy^2 + 29y - 24x^2 - 24y^2 \, dy$

$= \frac{19}{2} - \frac{1}{2} - 4z + 19y - \frac{2}{3}y^3 - 24y^2$

$\int_0^1 dy = \frac{19}{2} - \frac{1}{2} - \frac{1}{3} + \frac{19}{2} - \frac{1}{3} - \frac{1}{2} = \frac{693}{5}$

⑪  $F = 4z\mathbf{i} + 2z\mathbf{j} - x\mathbf{k}$

$z = -y - x - 1 \quad dx = -1$   
 $dy = -1$

$\int_0^1 \int_0^1 4z^2 + 2z + (-x) \, dx \, dy$

$= 4z^2 + 2z - \frac{x^2}{2} \Big|_0^1 = 4z + 1.5$

$\int_0^1 4z + 1.5 = \frac{4z}{2} + 1.5 = \frac{1}{2} + 1.5$

$4\pi - 4\pi e^{-4-1} = 4\pi(1 - e^{-5}) = \frac{11}{12}$

$2\pi \int_0^1 (1 - 10e^{-4-2}) = 2\pi(1 - 10e^{-6})$