16.4: 7, 13, 15, 19
16.5 5, 7, 9, 11

Ch16 HW

16.4

Orion Kress-Sanfilippo

7) $G(u,v) = (2u+v, u-4v, 3u)$ $\quad 0 \le u \le 1$, $0 \le v \le 1$

$T_u = \langle 2, 1, 3 \rangle \quad T_v = \langle 1, -4, 0 \rangle$

$$N(u,v) = \det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 1 & -4 & 0 \end{vmatrix} = (+12)\hat{i} + (+3)\hat{j} + (-9)\hat{k}$$

$$= \boxed{\langle 12, 3, -9 \rangle}$$

13) $G(u,v) = (u\cos v, u\sin v, u)$ $\quad 0 \le u \le 1$, $0 \le v \le 1$

$f(x,y,z) = z(x^2 + y^2)$

$z(u,v) = u$

$x = u\cos v \quad y = u\sin v$

$(x^2 + y^2) = 2u^2$

$r(u,v) = G(u,v)$

$r_u = \langle \cos(v), \sin(v), 1 \rangle$

$r_v = \langle -u\sin(v), u\cos(v)\ 0 \rangle$

$$|r_u \times r_v| = \sqrt{(u^2\cos^2(v)) + (u^2\sin^2(v)) + (u\cos^2(v) + u\sin^2(v))^2}$$

$$= \sqrt{(u^2(1)) + (u^2)} = u\sqrt{2}$$

Using Maple

$$\text{Ans} = \int_0^1 \int_0^1 u(2u^2)\, u\sqrt{2}\, du\, dv = \boxed{\frac{\sqrt{2}}{5}}$$

16.4 Cont

15) $y = 9 - z^2$ $\quad 0 \leq x \leq 3,\ 0 \leq z \leq 3$ $\quad f(x,y,z) = z$

$x = x \quad y = 9 - z^2 \quad z = z$

$r(x,z) = \langle x, 9-z^2, z \rangle$

$r_x = \langle 1, 0, 0 \rangle$

$r_z = \langle 0, -2z, 1 \rangle$

$|r_x \times r_z| = \sqrt{(0)^2 + (1)^2 + (2z)^2} = \sqrt{1+4z^2} = dS$

$= \sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2}$

Using Maple:

Ans: $\int_0^3 \int_0^3 z\sqrt{1+4z^2}\, dz\, dx = \boxed{\frac{3\sqrt{37}-1}{4}}$

19) $x^2 + y^2 = 4$ $\quad 0 \leq z \leq 4$ $\quad f(x,y,z) = e^{-z}$

$y = \sqrt{4-x^2}$ $\quad r(x,z) = \langle x, \sqrt{4-x^2}, z \rangle$

$0 \leq x \leq 2$ $\quad r_x = \left\langle 1, \frac{x}{\sqrt{4-x^2}}, 0 \right\rangle$

$r_z = \langle 0, 0, 1 \rangle$

$|r_x \times r_z| = \left(\frac{x}{\sqrt{4-x^2}}\right)\hat{i} - (1)\hat{j} + (0)\hat{k} = \sqrt{1 + \left(\frac{x}{\sqrt{4-x^2}}\right)^2} = \sqrt{1 + \frac{x^2}{4-x^2}}$

$= \sqrt{\frac{4 \cancel{-x^2} \cancel{+x^2}}{4-x^2}} = \frac{2}{\sqrt{4-x^2}}$

Using Maple

Ans: $\int_0^2 \int_0^4 e^{-z} \cdot \frac{2}{\sqrt{4-x^2}}\, dz\, dx = \boxed{(2-2e^{-4})2\pi}$

16.5 #s: 5, 7, 9, 11

16.5

5) $F = \langle y, z, x \rangle$ plane $3x - 4y + z = 1$ $\quad 0 \le x \le 1$, $0 \le y \le 1$ $\boxed{\text{Normal Up}}$

$\iint -Pg_x - Qg_y + R\,dA$

$z = 1 - 3x + 4y$ $\quad$ upwards normal = $z \ge 0$

$z_x = -3 \quad z_y = 4$

$$\text{Ans} = \int_0^1 \int_0^1 -(y \cdot -3) - ((1 - 3x + 4y) \cdot 4) + x \, dx\,dy$$

Using Maple

$$= \boxed{-4}$$

7) $F = \langle 0, 3, x \rangle$ $\quad x^2 + y^2 + z^2 = 9$

Spherical Coords: $x = 3\sin\phi\cos\theta \quad y = 3\sin\phi\sin\theta \quad z = 3\cos\phi$

$$r(\theta, \phi) = \langle 3\sin\phi\cos\theta, 3\sin\phi\sin\theta, 3\cos\phi \rangle$$

$$|r_\theta \times r_\phi| = \langle 9\sin^2\phi\cos\theta, 9\sin^2\phi\sin\theta, 9\sin\phi\cos\phi \rangle$$

$$0 \le \theta \le \frac{\pi}{2} \quad 0 \le \phi \le \frac{\pi}{2}$$

$$\text{Ans} = \int_0^{\pi/2} \int_0^{\pi/2} 0 + 3(9\sin^2\phi\sin\theta) + (3\sin\phi\cos\theta)(9\sin\phi\cos\phi)\, d\theta\, d\phi$$

Using Maple

$$= \frac{27\pi}{4} + 9 \approx 30.205$$

16.5 Cont

9) $F = \langle z, z, x \rangle$ $z = 9 - x^2 - y^2$

$0 \le x$
$0 \le y$
$0 \le z$

$z_x = -2x$ $z_y = -2y$

Ans: $$\int_0^3\int_0^3 +(9-x^2-y^2)\cdot(+2x) + (9-x^2-y^2)\cdot(+2y) + x \, dx \, dy$$

My Answer is Wrong!! $\boxed{94.5}$

11) $F = \langle y^2, 2, -x \rangle$ $x + y + z = 1$

$z = 1 - x - y$

$z_x = -1$ $z_y = -1$

Ans= $$\int_0^1\int_0^1 +y^2 + 2 - x \, dx \, dy = \boxed{\frac{11}{12}}$$