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## Exercise 16.4

Q7. calculate Tu, Tv, and N(u,v), find the equation of the tangent plane to the surface at that point.

$G(u,v) = (2u+v,\ u-4v,\ 3u)$ $\quad u=1,\ v=4.$

Answer: $T_u = (2, 1, 3)$ $\qquad G(1,4) = (6, -15, 3)$

$T_v = (1, -4, 0)$

$$T_u \times T_v = \begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ 1 & -4 & 0 \end{vmatrix} = 12i - j(-3) + (-\cancel{8}^{9})k$$

$$= (12, 3, -\cancel{8}^{9})$$

$$12(x-6) + 3(y+15) - \cancel{8}^{9}(z-3) = 0$$

$$4x + y - 3z\ \cancel{+8}\ -24 + 15 + \cancel{8}^{9} = 0$$

$$4x + y + (-3z) = \cancel{8}\,0$$

$$4x + y - 3z = \cancel{8}\,0$$

$\therefore N(u,v) = (12, 3, -9)$

$4x + y - 3z = 0$

Q13. calculate $\iint_S f(x,y,z)\,dS$ for the given surface.

$G(u,v) = (u\cos v,\ u\sin v,\ u)$ $\quad 0 \le u \le 1 \quad 0 \le v \le 1$

$f(x,y,z) = z(x^2+y^2)$

$G_u = \cos v\, i + \sin v\, j + 1k = (\cos v, \sin v, 1)$

$G_v = \cancel{-u\cos}\ -u\sin v\, i + u\cos v, 0k = (-u\sin v, u\cos v, 0)$

$$G_u \times G_v = \begin{vmatrix} i & j & k \\ \cos v & \sin v & 1 \\ -u\sin v & u\cos v & 0 \end{vmatrix} = (-u\cos v, -u\sin v, u).$$

$$|G_u \times G_v| = \sqrt{u^2\cos v^2 + u^2\sin v^2 + u^2}$$

$$= \sqrt{2}\,u$$

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$f(x,y,z) = z(x^2+y^2) = u\cdot((u\cos v)^2+(u\sin v)^2)$
$= u^3$

$\int_0^1\int_0^1 u^3\cdot\sqrt{2}\,u\,du\,dv = \frac{\sqrt{2}}{5}$

Q15. $y = 9-z^2,\ 0\le x\le 3,\ 0\le z\le 3,\ f(x,y,z)=z$

$z^2 = 9-y$ $\quad \frac{dz}{dx}=0 \quad \frac{dz}{dy} = -1\cdot\frac{1}{2}\cdot(9-y)^{-\frac{1}{2}}$

$z = \sqrt{9-y}$

$= -\frac{1}{2}\cdot(9-y)^{-\frac{1}{2}}$

$x\geq 0,\ y\geq 0$

$\cancel{ds = \sqrt{1+0^2+\left(-\frac{1}{2}(9-y)^{-\frac{1}{2}}\right)^2}}$

$\cancel{= \sqrt{1+\frac{9-x}{4}}}$

parametric representation of this surface is $(x, 9-z^2, z)$ $0\le x\le 3$, $0\le z\le 3$

$\cancel{\iint \sqrt{9-y}\cdot\sqrt{1+\frac{9-y}{4}}\cdot dA}$

$\cancel{= \int_0^{\frac{\pi}{2}}\int_0^3 \sqrt{9-r\sin\theta}\cdot\sqrt{1+\frac{9-r\sin\theta}{4}}}\cdot r\,dr\,d\theta$

$r(u,v) = (u, 9-v^2, v)$ $\quad 0\le u\le 3,\ 0\le v\le 3$

$ds = |r_u \times r_v|\,du\,dv = \sqrt{1+4v^2}\,du\,dv.$

$\int_0^3\int_0^3 v\,ds$

$= \int_0^3\int_0^3 v\cdot\sqrt{1+4v^2}\,du\,dv$

$= \frac{37\sqrt{37}-1}{4}$

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Exercise 16.5

Q7. compute $\iint_S F \cdot dS$

$F = (0, 3, x)$ part of sphere $x^2+y^2+z^2=9$

$x \geq 0,\ y \geq 0,\ z \geq 0$ outward-pointing normal.

Answer: $z = g(x,y) = \sqrt{9-x^2-y^2}$

$$\frac{dg}{dx} = -2x \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{9-x^2-y^2}} = \frac{-x}{\sqrt{9-x^2-y^2}}$$

$$\frac{dg}{dy} = \frac{-y}{\sqrt{9-x^2-y^2}}$$

$$\iint_S F \cdot dS = \iint_D \left(-0 \cdot \frac{dg}{dx} - 3\left(\frac{-y}{\sqrt{9-x^2-y^2}}\right) + x\right) dA$$

$$= \iint_D \frac{3y}{\sqrt{9-x^2-y^2}} + x \, dx\,dy$$

$$= \int_0^3 \int_0^{\sqrt{9-y^2}} \frac{3y}{\sqrt{9-x^2-y^2}} + x \quad dx\,dy$$

$$= 9 + \frac{27\pi}{4}$$

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Q5. $F = (y, z, x)$ plane $3x-4y+z=1$

$0 \le x \le 1 \quad 0 \le y \le 1$ upward pointing normal

$\iint_S F \cdot dS$

Ans: $g = z = 1-3x+4y$

$P = y, \quad Q = z \quad R = x$

$$\iint_S F \, dS$$
$$= \iint_D (-y\cdot(-3) - z\cdot 4 + x)\, dA$$
$$= \int_0^1 \int_0^1 (3y - 4z + x)\, dx\, dy$$
$$= \int_0^1 \int_0^1 (3y - 4(1-3x+4y) + x)\, dx\, dy$$
$$= \int_0^1 \int_0^1 3y - 4 + 12x - 16y + x \; dx\, dy$$
$$= \int_0^1 \int_0^1 13x - 13y - 4 \; dx\, dy$$
$$= \int_0^1 \frac{13}{2}x^2 - 13xy - 4x \Big|_0^1 dy$$
$$= \int_0^1 \frac{13}{2} - 13y - 4 \; dy$$
$$= \frac{5}{2}y - \frac{13}{2}y^2 \Big|_0^1$$
$$= \frac{5}{2} - \frac{13}{2}$$
$$= -\frac{8}{2}$$
$$= -4$$

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Q9. $F = (z, z, x)$ $\quad z = 9 - x^2 - y^2 \quad x \geq 0, y \geq 0$

$z \geq 0$ upward pointing normal.

Ans: $\quad z = g = 9 - x^2 - y^2$

$$\frac{dg}{dx} = -2x$$

$$x^2 = \cancel{z = 9 + y^2} \quad 9 - y^2 - z$$

$$\frac{dg}{dy} = -2y$$

$$\iint_S F \cdot dS = \iint_D (-2x \cdot -z - 2y \cdot -z + x)\, dA$$

$$= \int\int \cancel{(-2z(x-y) + x)\, dx\, dy}$$

$$2xz + 2zy + x \quad dA$$

$$= \int_0^3 \int_0^{\sqrt{9-y^2}} 2(9 - x^2 - y^2) \cdot (x + y) + x \, dx \, dy$$

$$= \frac{693}{5}$$

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Q11. $F = y^2 i + zj - xk$ portion of plane $x+y+z=1$ in the octant $x, y, z \geq 0$ upward pointing normal

Ans: $z = 1-x-y$ $\qquad \frac{dg}{dx} = -1 \qquad \frac{dg}{dy} = -1$

$g = 1-x-y$

$x = 1-y$

$$\iint_S F \cdot ds = \iint_D (-y^2 \cdot (-1) - z(-1) + (-x))\, dA$$

$$= \int_0^1 \int_0^{1-y} (y^2 + z - x)\, dx\, dy$$

$$= \frac{11}{12}$$