homework 11

16.4

7.) $G(u,v) = (2u+v,\ u-4v,\ 3u)$, $u=1$, $v=4$

$T_u = \frac{dx}{du}i + \frac{dy}{du}j + \frac{dz}{du}k = 2i + 1j + 3k$

$T_v = \frac{dx}{dv}i + \frac{dy}{dv}j + \frac{dz}{dv}k = 1i + -4j + 0$

$$T_u \times T_v = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -4 & 0 \end{vmatrix} = 12, 3, -9$$

$2(1)+4,\ 1-4(4),\ 3(1) = 6, -15, 3$

$(x-12)(6) + (y-3)(-15) + (z-3)(-9) = 0$

$6x - 72 + -15y + 45 - 9z + 27 = 0$

$6x - 15y - 9z = 0$ or $2x - 5y - 3z = 0$

13.) $G(u,v) = (u\cos v, u\sin v, u)$ $0 \le u \le 1$, $0 \le v \le 1$

$f(x,y,z) = z(x^2+y^2)$

$$\int_0^1 \int_0^1 u\left((u\cos v)^2 + (u\sin v)^2\right) du\,dv$$

$$\int_0^1 u\left((u\cos v)^2 + (u\sin v)^2\right) du \to \frac{(\sin^2(v)+\cos^2(v))u^4}{4} = \frac{u^4}{4}\Big|_0^1 = \frac{1}{4}$$

$$\int_0^1 \frac{1}{4}\,dv = \frac{1}{4}$$

15.) $y = 9 - z^2$, $0 \le x \le 3$, $0 \le z \le 3$ $f(x,y,z) = z$

$z^2 = 9 - y$

$z = \sqrt{9-y}$ $\quad 0 = \sqrt{9-y}$ $\quad y = 0$

$3 = \sqrt{9-y} \to 9 = 9 - y$ $\quad y = 0$

$0 = -\sqrt{9-y} = 0$

$3 = -\sqrt{9-y}$

$$\int_0^3 \int_0^3 z\,dx\,dz \qquad \int_0^3 3z\,dz =$$

19.) $x^2 + y^2 = 4$ $\quad 0 \le z \le 4$ $\quad f(x,y,z) = e^{-z}$

x, y $\quad 0 \le \theta \le 2\pi$

$$\int_0^{2\pi}\int_0^{2\pi}\int_0^4 x^2+y^2\,dz\,dx\,dz \qquad \iint_0^{\pi+\pi} A(x^2+y^2) = \frac{4\pi y^2 + 32\pi r^3}{3}$$

$\frac{128\pi^4}{3} = z$ $\quad e^{-z} = 4\pi(1-e^{-4})$

16.5

5.) $F = \langle y, z, x \rangle$ plane $3x - 4y + z = 1$

$0 \le x \le 1$, $0 \le y \le 1$

$3x = 1 - z + 4y$ $\quad -4y = 3x - 1 + z$

$x = \frac{1 - z + 4y}{3}$ $\quad z = -1 + 3x - 4y$ $\quad y = \frac{-3x + 1 - z}{4}$

$\frac{\partial}{\partial x} = 3$ $\quad \frac{\partial}{\partial y} = -4$ $\quad \frac{\partial}{\partial z} = 1$

$\sqrt{9 + 16} =$ 等 $\sqrt{25 + 1}\, dA = \sqrt{26}$

$\int_0^1 \int_0^1 3x - 4y + \sqrt{26}\, dA$

$\int_0^1 3x + \sqrt{26} - 2 \rightarrow \boxed{-4}$

7.) $F\langle 0, 3, x \rangle$ $x^2 + y^2 + z^2 = 9$

$\iint_S F\, ds \rightarrow \iint_D -P\left(\frac{\partial z}{\partial x}\right) - Q\left(\frac{\partial z}{\partial y}\right) + R\, dA$

$\iint_D -0 - 3(2z) + x = \iint_D -6z + x\, dA$

$z = \sqrt{9 - x^2 - y^2}$ $\quad -6\left(\sqrt{9 - x^2 - y^2}\right) + x\, dA =$

$\iint_D -6\sqrt{9 - x^2 - y^2} + x\, dA = \frac{27}{12}(3\pi + A)$

9.) $P = 2$ $Q = 2$

$-2(2x) - 2(2y) + x =$

$-4x - 4y + x = -3x - 4y \approx 130.6$

11.) $y^2 i + 2j - xk$ $\quad x + y + z = 1$

$-2y(1) - 0 + x = -2y + x\, dA$

$\iint_D -2y + x\, dA \rightarrow \boxed{\frac{11}{12}}$