16.4 : 7, 13, 15, 19

7) $G(u,v) = (2u+v,\ u-4v,\ 3u)$; $u=1,\ v=4$

$x = 2u+v \quad y = u-4v \quad z = 3u$

$r = (2u+v)i + (u-4v)j + (3u)k$

$r_u = 2i + j + 3k \quad \langle 2, 1, 3\rangle$

$r_v = i - 4 \cdot j \quad \langle 1, -4, 0\rangle$

$$r_u \times r_v = \begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ 1 & -4 & 0 \end{vmatrix} = (0+12)i - (0-3)j + (-8-1)k$$

$= 12i + 3j - 9k \rightarrow \langle 12, 3, -9\rangle$

$= 3\langle 4, 1, -3\rangle$

$(2+4,\ 1-16,\ 3)$

$\rightarrow (6, -15, 3)$

$4(x-6) + 1(y+15) - 3(z-3) = 0$

$4x - 24 + y + 15 - 3z + 9 = 0$

$4x + y - 3z = 0$

13) $\iint_S f(x,y,z)\, dS$; $G(u,v) = (u\cos v, u\sin v, u)$, $0 \le u \le 1$, $0 \le v \le 1$; $f(x,y,z) = z(x^2+y^2)$

$P = u\cos v \quad Q = u\sin v \quad R = u$

$f(G(u,v)) = u((u\cos v)^2 + (u\sin v)^2) = u^3$

$\frac{\partial G}{\partial u} = (\cos v, \sin v, 1)$

$\times$

$\frac{\partial G}{\partial v} = (-u\sin v, u\cos v, 0)$

$$= \begin{vmatrix} i & j & k \\ \cos v & \sin v & 1 \\ -u\sin v & u\cos v & 0 \end{vmatrix} = (0 - u\cos v)i - (0 + u\sin v)j + (u\cos^2 v + u\sin^2 v)k$$

$\langle -u\cos v, -u\sin v, u\rangle$

$\sqrt{(-u\cos v)^2 + (-u\sin v)^2 + u^2} = \sqrt{u^2(\cos^2 + \sin^2 + 1)} = u\sqrt{2}$

$$\int_0^1\int_0^1 u\sqrt{2}(u^3)\, du\, dv = \frac{\sqrt{2}}{5}$$

15) $\iint_S f(x,y,z)\, dS$; $y = 9 - z^2$, $0 \le x \le 3$, $0 \le z \le 3$; $f(x,y,z) = z$

$f(G(u,v)) = \langle u, 9-v^2, v\rangle \quad 0 \le u \le 3,\ 0 \le v \le 3$

$f_u = \langle 1, 0, 0\rangle \times f_v = \langle 0, -2v, 1\rangle$

$$= \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & -2v & 1 \end{vmatrix} = (0-0)i - (1-0)j + (-2v-0)k$$

$= \langle 0, -1, -2v\rangle$

$\sqrt{0 + 1 + 4v^2} = \sqrt{1+4v^2}\, du\, dv$

$$\int_0^3\int_0^3 v\sqrt{1+4v^2}\, du\, dv$$

$$= \frac{37\sqrt{37} - 1}{4} \approx 56.02$$

19) $\iint_S f(x,y,z)\, dS$; $x^2 + y^2 = 4$, $0 \le z \le 4$; $f(x,y,z) = e^{-z}$

$G(\theta, z) = (\overset{x}{2\cos\theta}, \overset{y}{2\sin\theta}, \overset{z}{z})$ $\quad \{(\theta, z) \mid 0 \le \theta \le 2\pi,\ 0 \le z \le 4\}$

$T_\theta = \langle -2\sin\theta, 2\cos\theta, 0\rangle$

$T_z = \langle 0, 0, 1\rangle$

$$\times \begin{vmatrix} i & j & k \\ -2\sin & 2\cos & 0 \\ 0 & 0 & 1 \end{vmatrix} = (2\cos\theta - 0)i - (-2\sin\theta + 0)j + (0-0)k$$

$\langle 2\cos\theta, 2\sin\theta, 0\rangle$

$\sqrt{4\cos^2 + 4\sin^2 + 0} = 2$

$$\int_0^{2\pi}\int_0^4 2e^{-z}\, d\theta\, dz$$

$= 4\pi(1 - e^{-4})$

16.5 : 5, 7, 9, 11

5) $\iint_S F\cdot ds$; $F=\langle y,z,x\rangle$, plane $3x-4y+z=1$, $0\le x\le 1$, $0\le y\le 1$, upward pointing normal

$G(x,y)=\langle x,y,1-3x+4y\rangle$, $0\le x,y\le 1$

$T_x=\langle 1,0,-3\rangle \times T_y\langle 0,1,4\rangle$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & -3 \\ 0 & 1 & 4 \end{vmatrix} = (0+3)i-(4-0)j+(1-0)k$$

$\langle 3,-4,1\rangle$

$F\cdot n = \langle y,1-3x+4y,x\rangle\cdot\langle 3,-4,1\rangle$

$= 13x-13y-4$

$\int_0^1\int_0^1 13x-13y-4\,dx\,dy$

$= -4$

7) $\iint_S F\cdot ds$; $F=\langle 0,3,x\rangle$, part of sphere $x^2+y^2+z^2=9$, where $x\ge 0, y\ge 0, z\ge 0$, outward pointing normal

$G(\theta,\phi)=(3\cos\theta\sin\phi, 3\sin\theta\sin\phi, 3\cos\phi)$, $0\le\theta\le\frac{\pi}{2}$, $0\le\phi\le\frac{\pi}{2}$

$T_\theta=\frac{\partial G}{\partial\theta}=\langle -3\sin\theta\sin\phi, 3\cos\theta\sin\phi, 0\rangle \times T_\phi=\langle 3\cos\theta\cos\phi, 3\sin\theta\cos\phi, -3\sin\phi\rangle$

$$\begin{vmatrix} i & j & k \\ -3\sin\theta\sin\phi & 3\cos\theta\sin\phi & 0 \\ 3\cos\theta\cos\phi & 3\sin\theta\cos\phi & -3\sin\phi \end{vmatrix} = \langle -9\cos\theta\sin^2\phi, 9\sin\theta\sin^2\phi, 9\sin\phi\cos\phi\rangle$$

$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\langle 0,3,3\cos\theta\sin\phi\rangle\cdot$
$\langle -9\cos\theta\sin^2\phi, 9\sin\theta\sin^2\phi, 9\sin\phi\cos\phi\rangle$

$F(G(\theta,\phi))=\langle 0,3,3\cos\theta\sin\phi\rangle$

$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} 27\sin\theta\sin^2\phi+27\cos\theta\sin^2\phi\cos\theta\,d\theta\,d\phi$

$\frac{27\pi}{4}+27\int_0^{\frac{\pi}{2}}\sin^2\phi\cos\phi\,d\phi$

$= \frac{27}{12}(3\pi+4)$

9) $\iint_S F\cdot ds$; $F=\langle z,z,x\rangle$, $z=9-x^2-y^2$, $x\ge 0, y\ge 0, z\ge 0$, upward pointing normal

$G(r,\theta)=\langle r\cos\theta, r\sin\theta, 9-r^2\rangle$ $\{(r,\theta)\mid 0\le r\le 3, 0\le\theta\le\frac{\pi}{2}\}$

$T_r=\langle\cos\theta,\sin\theta,-2r\rangle \times T_\theta=\langle -r\sin\theta,\cos\theta,0\rangle$

$$\begin{vmatrix} i & j & k \\ \cos\theta & \sin\theta & -2r \\ -r\sin\theta & \cos\theta & 0 \end{vmatrix} = \langle 2r^2\cos\theta, 2r\sin\theta, r\rangle$$

$\times$

$F(G(r,\theta))=\langle 9-r^2, 9-r^2, r\cos\theta\rangle$

$\int_0^{\frac{\pi}{2}}\int_0^3 2r^2(9-r^2)(\cos\theta+\sin\theta)+r^2\cos\theta\,dr\,d\theta$

$= \frac{693}{5}$

11) $\iint_S F\cdot ds$; $F=y^2i+2j-xk$, portion of the plane $x+y+z=1$ in the $x,y,z\ge 0$; upward normal

$G(u,v)=\langle u,v,1-u,v\rangle$

$\int_0^1\int_0^{1-v} v^2+2-u\,du\,dv$

$T_u=\langle 1,0,-1\rangle\times T_v=\langle 0,1,-1\rangle$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{vmatrix} = \langle 1,1,1\rangle$$

$\int_0^1 v^2-v^3+2-2v-\frac{(1-v)^2}{2}\,dv$

$F(G(u,v))\cdot\langle 1,1,1\rangle$

$\langle v^2,2,-u\rangle\cdot\langle 1,1,1\rangle$

$= v^2+2-u$

$= \frac{11}{12}$