11/21/20. 16.4 Parametrerized Surface and Surface Integrals

16.4: 7, 13, 15, 19.

7) $G(u,v) = (2u+v, u-4v, 3u)$

$T_u = \langle 2, 1, 3 \rangle$

$T_v = \langle +1, -4, 0 \rangle$

$T_u \times T_v$

$$\begin{pmatrix} i & j & k \\ 2 & 1 & 3 \\ 1 & -4 & 0 \end{pmatrix} = \langle 12, 3, -9 \rangle$$

$G(1,4) = \langle 6, -15, 3 \rangle$

$12(x-6) + 3(y+15) - 9(z-3) = 0.$

13) $G(u,v) = (u\cos v, u\sin v, u)$ $[0,1] \to u, v.$

$f(x,y,z) = z(x^2+y^2)$

$G_u = \langle \cos v, \sin v, 1 \rangle$

$G_v = \langle -u\sin v, u\cos v, 0 \rangle$

$\|G_u \times G_v\| = u\sqrt{2}$

$$\int_0^1 \int_0^1 \sqrt{2}\, u^4 \, du\, dv = \sqrt{2} \int_0^1 \frac{1}{5} u^5 \Big|_0^1 = \frac{\sqrt{2}}{5}$$

15) $y = 9 - z^2 \quad 0 \le x \le 3 \;, 0 \le z \le 3,$

$f(x,y,z) = z.$

$G(x,z) = (x, 9-z^2, z)$

$f(G(x,z)) = z$

$G_x = \langle 1,0,0 \rangle \quad G_z \; \langle 0, -2z, 1 \rangle.$

$G_x \times G_z$

$$\begin{pmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & -2z & 1 \end{pmatrix} = \langle 0, -1, -2z \rangle$$

$\| mag \| = \sqrt{1+4z^2}$

$$\int_0^3 \int_0^3 z\sqrt{1+4z^2} \, dx \, dz = 56.02.$$

19) $x^2+y^2=4, \quad 0 \le z \le 4,$

$f(x,y,z) = e^{-z}$

$x = 2\cos\theta \quad y = 2\sin\theta \quad z = z.$

$G(\theta, z) = \langle 2\cos\theta, 2\sin\theta, z \rangle$

$G_\theta = \langle -2\sin\theta, 2\cos\theta, 0 \rangle, \quad G_z = \langle 0,0,1 \rangle.$

$G_\theta \times G_z.$

$$\begin{pmatrix} i & j & k \\ -2\sin\theta & 2\cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$= \langle 2\cos\theta, 2\sin\theta, \rangle$

$f(G(\theta, z)) = e^{-z}$

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$\iint f(\sigma(u,v))$

$$\int_0^{2\pi}\int_0^{4} 2e^{-z}\, d\theta\, dz = 4\pi\left(1-\frac{1}{e^{4}}\right)$$

11/21/20 16.5 Surface Intgrals of Vector Fields

16.5: 5, 7, 9, 11

5) $F = \langle y, z, x \rangle$ plane $3x - 4y + z = 1$.
$x: [0,1]$ $z = 1 - 3x - 4y$.
$y: [0,1]$
$\iint_D \left(-P\frac{dg}{dx} - Q\frac{dg}{dy} + R\right) dA.$

$g(x,y) = 1 - 3x + 4y$
$g_x = -3$
$g_y = 4.$

$\iint_D (-y(-3) - z(4) + x)) dA$
$\iint 3y - (1 - 3x + 4y)(4) + x) dA$
$\iint 13x - 13y - 4 \, dA$
$\int_0^1 \int_0^1 (13x - 13y - 4) dx\, dy = -4.$

7) $F = \langle 0, 3, x \rangle$ in the sphere $x^2 + y^2 + z^2 = 1$
$\frac{1}{2} \le z \le \frac{\sqrt{3}}{2}$
$F(\theta, \phi) = (3\cos\theta \sin\phi, 3\sin\theta \sin\phi, 3\cos\phi)$
$0 \le \theta \le \frac{\pi}{2}$ $0 \le \phi \le \frac{\pi}{2}$
$F(G(\phi)) = \langle 0, 3, 3\cos\theta \sin\phi \rangle$
$G_\theta = \langle -3\sin\theta \sin\phi, 3\cos\theta \sin\phi, 0 \rangle$
$G_\phi = \langle 3\cos\theta \cos\phi, 3\sin\theta \cos\phi, -3\sin\phi \rangle,$

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$G_\theta \times G_\phi = \langle -9\cos\theta\sin\phi, -9\sin\theta\cos\phi\sin\phi, -9\sin^2\theta\cos\phi$

$-9\cos^2\theta \sin\theta\cos\phi$

$$\int_0^{\pi/2}\int_0^{\pi/2} (27\sin\theta\sin^2\phi + 27\cos\theta\sin^2\phi\cos\phi)\,d\phi\,d\theta$$

$$= \frac{27}{12}(3\pi+4).$$

9) $F = \langle z, z, x \rangle$

$z = 9 - x^2 - y^2$

$x \geq 0, y \geq 0, z \geq 0.$

$g(x,y) = (x, y, 9-x^2-y^2)$

$g_x = \langle 1, 0, -2x \rangle$

$g_y = \langle 0, 1, -2y \rangle.$

$$g_x \times g_y = \begin{pmatrix} i & j & k \\ 1 & 0 & -2x \\ 0 & 1 & -2y \end{pmatrix}$$

$\langle -2x, 2y, 1 \rangle$

$F(g(x,y)) = \langle 9-x^2-y^2, 9-x^2-y^2, x \rangle.$

$\iint F(g(x,y)) \cdot n(x,y)\, dA.$

$\iint 19x + 18y - 2xy(x^2+y^2)\, dA$

$x = r\cos\theta \quad y = r\sin\theta.$

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11) $F = y^2 i + 2j - xk$. $\qquad x+y+z=1$.

$\langle y^2, 2, -x \rangle$.

$g(x,y) = (x, y, 1-x-y)$

$F(g(x,y)) = \langle y^2, 2, x \rangle$

$g_x = \langle 1, 0, -1 \rangle$

$g_y = \langle 0, 1, -1 \rangle$

$g_x \times g_y \begin{pmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix} = \langle 1, 1, 1 \rangle$.

Mag $= \sqrt{3}$

$F(g(x,y)) = \langle y^2, 2, x \rangle$

$\iint \langle r^2 \sin^2\theta, 2, r\cos\theta \rangle \cdot \langle 1, 1, 1 \rangle \, dA$

$\int_0^1 \int_0^{\pi/2} (r^2 \sin^2\theta + 2 + r\cos\theta) \, d\theta \, dr = 11/12$

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