16.4 7, 13, 15, 19
16.5 5, 7, 9, 11

16.4

7) $G(u,v) = (2u+v,\ u-4v,\ 3u)$ $\quad u=1 \quad v=4$

$r(u,v) = (2u+v)\hat{i} + (u-4v)\hat{j} + 3u\,\hat{k}$
$r_u = 2i + 1j + 3\hat{k} = T_u$
$r_v = i + -4j + 0\hat{k} = T_v$
$N(u,v) = r_u \times r_v = 3\langle 4, 1, -3\rangle$
$P = G(1,4) = \langle 6, -15, 3\rangle$

$4(x-6) + (y+15) - 3(z-3) = 0$

13) $\iint_S f(x,y,z)\, dS$

$G(u,v) = (u\cos v,\ u\sin v,\ u)$, $\quad 0 \le u \le 1$, $\quad 0 \le v \le 1$
$f(x,y,z) = z(x^2+y^2)$

$\int_0^1 \int_0^1 u((u\cos v)^2 + (u\sin v)^2) \cdot \sqrt{2}\, u \, du\,dv = \frac{\sqrt{2}}{5}$

$N(u,v) = G_u \times G_v = \langle \cos v, \sin v, 1\rangle \times \langle -u\sin v, u\cos v, 0\rangle =$

$\langle -u\cos v, -u\sin v, u\rangle \to \|N\| = \sqrt{2}\,u$

15) $y = 9 - z^2 \quad 0 \le x \le 3 \quad 0 \le z \le 3 \quad F(x,y,z) = z$

$f(G(u,v)) = \langle u, 9-v^2, v\rangle$

$T_u = \langle 1, 0, 0\rangle \qquad N = T_u \times T_v = \langle 0, -1, -2v\rangle$
$T_v = \langle 0, -2v, 1\rangle \qquad \|N\| = \sqrt{1+4v^2}$

$\int_0^3 \int_0^3 v\sqrt{1+4v^2}\, du\,dv = \frac{37\sqrt{37}-1}{4} \approx 56.02$

19) $x^2+y^2=4$, $\quad 0 \le z \le 4$; $\quad F(x,y,z) = e^{-z}$

$G(\theta, z) = (2\cos\theta, 2\sin\theta, z) \quad 0 \le \theta \le 2\pi \quad 0 \le z \le 4$

$T_\theta = \langle -2\sin\theta, 2\cos\theta, 0\rangle \quad T_z = \langle 0, 0, 1\rangle \quad N = T_\theta \times T_z = \langle 2\cos\theta, 2\sin\theta, 0\rangle$

$\|N\| = 2 \to \int_0^{2\pi} \int_0^4 2e^{-z}\, d\theta\, dz = 4\pi(1-e^{-4})$

16.5 5, 7, 9, 11

5) $F = \langle y, z, x \rangle$, plane $3x - 4y + z = 1$, $0 \le x \le 1$, $0 \le y \le 1$, upward

$G(x,y) = \langle x, y, 1 - 3x + 4y \rangle \quad 0 \le x, y \le 1$

$T_x = \langle 1, 0, -3 \rangle \qquad n(x,y) = T_x \times T_y = \langle 3, -4, 1 \rangle$
$T_y = \langle 0, 1, 4 \rangle$

$$\int_0^1 \int_0^1 F(G(x,y)) \cdot n(x,y)\, dx\, dy = \int_0^1 \int_0^1 13x - 13y - 4\, dx\, dy = -4$$

7) $F = \langle 0, 3, x \rangle \quad x^2 + y^2 + z^2 = 9 \quad x \ge 0,\ y \ge 0,\ z \ge 0$ outward

$G(\theta, \phi) = (3\cos\theta \sin\phi, 3\sin\theta \sin\phi, 3\cos\phi), \quad 0 \le \theta \le \pi/2 \quad 0 \le \phi \le \pi/2$

$T_\theta = \langle -3\sin\theta \sin\phi, 3\cos\theta \sin\phi, 0 \rangle \qquad \vec{n} = T_\theta \times T_\phi = \langle -9\cos\theta \sin^2\phi, -9\sin\theta \sin^2\phi, -9\sin\phi\cos\phi \rangle$
$T_\phi = \langle 3\cos\theta\cos\phi, 3\sin\theta\cos\phi, -3\sin\phi \rangle$

$F(G(\theta, \phi)) = \langle 0, 3, 3\cos\theta\sin\phi \rangle$

$$\int_0^{\pi/2} \int_0^{\pi/2} \vec{F}(G(\theta,\phi)) \cdot \vec{n}\, d\theta\, d\phi = \frac{27}{12}(3\pi + 4)$$ * Use maple

9) $F = \langle z, z, x \rangle \quad z = 9 - x^2 - y^2 \quad x \ge 0,\ y \ge 0,\ z \ge 0$ upward

$G(r, \theta) = \langle r\cos\theta, r\sin\theta, 9 - r^2 \rangle \quad 0 \le r \le 3, \quad 0 \le \theta \le \pi/2$

$T_r = \langle \cos\theta, \sin\theta, -2r \rangle \quad T_\theta = \langle -r\sin\theta, r\cos\theta, 0 \rangle \quad N = \langle 2r^2\cos\theta, 2r^2\sin\theta, r \rangle$

$$\int_0^{\pi/2} \int_0^3 \vec{F}(G(r,\theta)) \cdot \vec{n}\, dr\, d\theta = \frac{693}{5}$$ * use maple

11) $F = y^2 i + 2j - xk \quad x + y + z = 1$ in the octant $x, y, z \ge 0$

$G(u,v) = \langle u, v, 1 - u - v \rangle$
$F(G(u,v)) = \langle v^2, 2, -u \rangle$
$T_u = \langle 1, 0, -1 \rangle$
$T_v = \langle 0, 1, -1 \rangle$
$n = \langle 1, 1, 1 \rangle$

$$\int_0^1 \int_0^1 F(G(u,v)) \cdot \vec{n}\, du\, dv = \frac{11}{12}$$

Use maple