

16.4 7, 13, 15, 19

16.5 5, 7, 9, 11

16.4

$$7) G(u, v) = (2u + v, u - 4v, 3u) \quad u=1 \quad v=4$$

$$r(u, v) = (2u + v)\hat{i} + (u - 4v)\hat{j} + 3u\hat{k}$$

$$r_u = 2\hat{i} + \hat{j} + 3\hat{k} = T_u$$

$$r_v = \hat{i} - 4\hat{j} + 0\hat{k} = T_v$$

$$N(u, v) = r_u \times r_v = 3\langle 4, 1, -3 \rangle$$

$$P = 6(1, 4) = \langle 6, -15, 3 \rangle$$

$$4(x-6) + (y+15) - 3(z-3) = 0$$

$$13) \iint_S f(x, y, z) \, dS$$

$$G(u, v) = (u \cos v, u \sin v, u), \quad 0 \leq u \leq 1, \quad 0 \leq v \leq 1$$

$$f(x, y, z) = z(x^2 + y^2)$$

$$\int_0^1 \int_0^1 u((u \cos v)^2 + (u \sin v)^2) \cdot \sqrt{2} u \, du \, dv = \frac{\sqrt{2}}{5}$$

$$N(u, v) = G_u \times G_v = \langle \cos v, \sin v, 1 \rangle \times \langle -u \sin v, u \cos v, 0 \rangle =$$

$$\langle -u \cos v, -u \sin v, u \rangle \rightarrow \|N\| = \sqrt{2} u$$

$$15) y = 9 - z^2 \quad 0 \leq x \leq 3 \quad 0 \leq z \leq 3 \quad f(x, y, z) = z$$

$$f(G(u, v)) = \langle u, 9 - v^2, v \rangle$$

$$T_u = \langle 1, 0, 0 \rangle \quad N = T_u \times T_v = \langle 0, -1, -2v \rangle$$

$$T_v = \langle 0, -2v, 1 \rangle \quad \|N\| = \sqrt{1+4v^2}$$

$$\int_0^3 \int_0^3 v \sqrt{1+4v^2} \, du \, dv = \frac{37\sqrt{37} - 1}{4} \approx 56.02$$

$$19) x^2 + y^2 = 4, \quad 0 \leq z \leq 4; \quad f(x, y, z) = e^{-z}$$

$$G(\theta, z) = (2 \cos \theta, 2 \sin \theta, z) \quad 0 \leq \theta \leq 2\pi \quad 0 \leq z \leq 4$$

$$T_\theta = \langle -2 \sin \theta, 2 \cos \theta, 0 \rangle \quad T_z = \langle 0, 0, 1 \rangle \quad N = T_\theta \times T_z = \langle 2 \cos \theta, 2 \sin \theta, 0 \rangle$$

$$\|N\| = 2 \rightarrow \int_0^{2\pi} \int_0^4 2e^{-z} \, d\theta \, dz = 4\pi(1 - e^{-4})$$

16.5 5, 7, 9, 11

5) $F = \langle y, z, x \rangle$, plane $3x - 4y + z = 1$, $0 \leq x \leq 1$, $0 \leq y \leq 1$, upward

$$G(x, y) = \langle x, y, 1 - 3x + 4y \rangle \quad 0 \leq x, y \leq 1$$

$$T_x = \langle 1, 0, -3 \rangle \quad T_y = \langle 0, 1, 4 \rangle \quad n(x, y) = T_x \times T_y = \langle 3, -4, 1 \rangle$$

$$\int_0^1 \int_0^1 F(G(x, y)) \cdot n(x, y) \, dx \, dy = \int_0^1 \int_0^1 (3x - 13y - 4) \, dx \, dy = -4$$

7) $F = \langle 0, 3, x \rangle$ $x^2 + y^2 + z^2 = 9$ $x \geq 0, y \geq 0, z \geq 0$ outward

$$G(\theta, \phi) = \langle 3 \cos \theta \sin \phi, 3 \sin \theta \sin \phi, 3 \cos \phi \rangle, \quad 0 \leq \theta \leq \pi/2, \quad 0 \leq \phi \leq \pi/2$$

$$T_\theta = \langle -3 \sin \theta \sin \phi, 3 \cos \theta \sin \phi, 0 \rangle \quad T_\phi = \langle 3 \cos \theta \cos \phi, 3 \sin \theta \cos \phi, -3 \sin \phi \rangle$$
$$\vec{n} = T_\theta \times T_\phi = \langle 9 \cos \theta \sin^2 \phi, 9 \sin \theta \sin^2 \phi, 9 \sin \theta \cos \phi \rangle$$

$$F(G(\theta, \phi)) = \langle 0, 3, 3 \cos \theta \sin \phi \rangle$$

$$\int_0^{\pi/2} \int_0^{\pi/2} F(G(\theta, \phi)) \cdot \vec{n} \, d\theta \, d\phi = \frac{27}{12} (3\pi + 4) \quad * \text{ Use maple}$$

9) $F = \langle z, z, x \rangle$ $z = 9 - x^2 - y^2$ $x \geq 0, y \geq 0, z \geq 0$ upward

$$G(r, \theta) = \langle r \cos \theta, r \sin \theta, 9 - r^2 \rangle \quad 0 \leq r \leq 3, \quad 0 \leq \theta \leq \pi/2$$

$$T_r = \langle \cos \theta, \sin \theta, -2r \rangle \quad T_\theta = \langle -r \sin \theta, r \cos \theta, 0 \rangle \quad N = \langle 2r^2 \cos \theta, 2r^2 \sin \theta, r \rangle$$

$$\int_0^{\pi/2} \int_0^3 F(G(r, \theta)) \cdot \vec{n} \, dr \, d\theta = \frac{693}{5} \quad * \text{ Use maple}$$

11) $F = y^2 \mathbf{i} + z \mathbf{j} - x \mathbf{k}$ $x + y + z = 1$ in the octant $x, y, z \geq 0$

$$G(u, v) = \langle u, v, 1 - u - v \rangle \quad \int_0^1 \int_0^1 F(G(u, v)) \cdot \vec{n} \, du \, dv = \frac{11}{12}$$

$$F(G(u, v)) = \langle v^2, 1 - u - v, -u \rangle$$

$$T_u = \langle 1, 0, -1 \rangle$$

$$T_v = \langle 0, 1, -1 \rangle$$

$$n = \langle 1, 1, 1 \rangle$$

Use Maple