16.4

7. $G(u,v)=(2u+v,\ u-4v,\ 3u)$; $u=1,\ v=4$

$r_u=2i+j+3k$, $r_v=i-4j+0k$

$r_u=(2,1,3)$, $r_v=(1,-4,0)$

$$r_u\times r_v=\begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ 1 & -4 & 0\end{vmatrix}=(12,3,-9)=3(4,1,-3)$$

$G(1,4)=(6,-15,3)$

$4(x-6)+(y+15)\cancel{*}-3(z-3)=0$

13. $G(u,v)=(u\cos v,\ u\sin v,\ u)$, $0\le u\le 1$, $0\le v\le 1$

$f(x,y,z)=z(x^2+y^2)$

$$\iint_S f(x,y,z)\,dS=\iint_D f(G(u,v))\cdot\|\vec{G_u}\times G\vec{v}\|\,dA$$

$G_u=(\cos v,\ \sin v,\ 1)$ $\quad G_v=(-u\sin v,\ u\cos v,\ 0)$

$G_u\times G_v=(-u\cos v,\ -u\sin v,\ u\cos^2 v+u\sin^2 v)$

$\|G_u\times G_v\|=\sqrt{u^2\cos^2 v+u^2\sin^2 v+u^2}=\sqrt{u^2+u^2}=\sqrt{2}u$

$$\int_0^1\int_0^1 u(u^2\cos^2 v+u^2\sin^2 v)\cdot\sqrt{2}u\,dv\,du$$

$$=\sqrt{2}\int_0^1\int_0^1 u^2\cdot u^2\,dv\,du=\sqrt{2}\cdot\frac{1}{5}=\frac{\sqrt{2}}{5}$$

15. $y=9-z^2$, $0\le x\le 3$, $0\le z\le 3$, $f(x,y,z)=z$

$y_x=0$, $y_z=-2z$

$dS=\sqrt{1+2z^2}\,dA$

$$=\int_0^3\int_0^3 z\sqrt{1+2z^2}\,dz\,dx$$

$$=\frac{37\sqrt{37}-1}{4}$$

19. $x^2+y^2=4$, $0\le z\le 4$, $f(x,y,z)=e^{-z}$

$x=2\cos u$, $y=2\sin u$, $0\le u\le 2\pi$

Let $z=v$, $0\le v\le 4$

$r(u,v)=\langle 2\cos u, 2\sin u, v\rangle$

$\iint_S f(x,y,z)\,dS=\iint_D f(r(u,v))\,|r_u\times r_v|\,dA$

$f(r(u,v))=e^{-v}$

$r_u=\langle -2\sin u, 2\cos u, 0\rangle$

$r_v=\langle 0,0,1\rangle$

$r_u\times r_v=2\cos u\,i+2\sin u\,j+0k$

$|r_u\times r_v|=2$

$\int_0^{2\pi}\int_0^4 2e^{-v}\,dv\,du$

$=4\pi(1-e^{-4})$

16.5

5. $F=(y,z,x)$ $3x-4y+z=1$, $0\le x\le 1, 0\le y\le 1$

$z=1+4y-3x$

$r(x,y)=xi+yj+(1+4y-3x)k$

$r_x=i-3k$, $r_y=j+4k$

$n=3i-4j+k$

$\iint_D(3y-4z+x)\,dA$

$\iint_D(3y+x)\,dx\,dy=-4$

7. $F=(0,3,x)$ $x^2+y^2+z^2=9$, $x\geq 0, y\geq 0, z\geq 0$

$r(\phi,\theta)=\langle 3\sin\phi\cos\theta, 3\sin\phi\sin\theta, 3\cos\phi\rangle$

$r_\phi=\langle 3\cos\theta\cos\phi, 3\sin\theta\cos\phi, -3\sin\phi\rangle$

$r_\theta=\langle -3\sin\theta\sin\phi, 3\cos\theta\sin\phi, 0\rangle$

$r_\phi\times r_\theta=\langle 9\cos\theta\sin^2\phi, 9\sin\theta\sin^2\phi, 9\sin\phi\cos\phi\rangle$

$F\cdot(r_\phi\times r_\theta)=\langle 0,3,x\rangle\cdot\langle 9\cos\theta\sin^2\phi, 9\sin\theta\sin^2\phi, 9\sin\phi\cos\phi\rangle$

$=(27\sin\theta\sin^2\phi+27\sin^2\phi\cos\phi\cos\theta)$

$\iint_S F\cdot dS=\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} F\cdot(r_\phi\times r_\theta)\,d\theta\,d\phi$

$=\frac{27\pi}{4}+9$

9. $F=\langle z,z,x\rangle$, $z=9-x^2-y^2$, $x\geq 0, y\geq 0, z\geq 0$

$R(x,y)=\langle x, y, 9-x^2-y^2\rangle$

$R_x=\langle 1,0,-2x\rangle$, $R_y=\langle 0,1,-2y\rangle$

$R_x\times R_y=\langle 2x,2y,1\rangle$

$\iint_D \langle 2,2,x\rangle\langle 2x,2y,1\rangle\,dA$

$=\iint_D 2(9-x^2-y^2)(x+y)+x\,dA$

$=\frac{693}{5}$

11. $F=y^2i+2j-xk$, $x+y+z=1$

$-\iint_D (y^2+2-x)\,dy\,dx$

$\int_0^1\int_0^{1-x}(x-y^2-2)\,dy\,dx=\frac{11}{12}$