16.4 Homework

⑦ $T = (2u+v,\ u-4v,\ 3u)$

$\to T_u = \langle 2,1,3\rangle,\quad T_v = \langle 1,-4,0\rangle$

$\to \langle 2,1,3\rangle \times \langle 1,-4,0\rangle$

$\to N(u,v) = \langle 12,3,-9\rangle$

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⑬ $G(u,v) = (u\cos v,\ u\sin v,\ u),\quad u:[0,1],\quad v:[0,1]$

$\to f(x,y,z) = z(x^2+y^2)$

$\to G_u = \langle \cos v,\ \sin v,\ 1\rangle,\quad G_v = \langle -u\sin v,\ u\cos v,\ 0\rangle$

$\to G_u \times G_v = \sqrt{2}\,u$

$\to \int_0^1\int_0^1 \sqrt{2}\,u^4\,du\,dv = \frac{\sqrt{2}}{5}$

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⑮ $y = 9-z^2,\quad 0 \le x \le 3,\quad 0 \le z \le 3;\quad f(x,y,z) = z$

$\to G(x,z) = (x,\ 9-z^2,\ z)$

$\to f(G(x,z)) = z$

$\to G_x = \langle 1,0,0\rangle,\quad G_z = \langle 0,-2z,1\rangle$

$\to \int_0^3\int_0^3 z\sqrt{1+4z^2}\,dx\,dz = \frac{1}{4}(37\sqrt{37}-1)$

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⑲ $x^2+y^2 = 4,\quad 0 \le z \le 4,\quad f(x,y,z) = e^{-z}$

$\to x = 2\cos\theta,\quad y = 2\sin\theta,\quad z = z$

$\to G(\theta,z) = (2\cos\theta,\ 2\sin\theta,\ z)$

$\to \int_0^{2\pi}\int_0^4 2e^{-z}\,d\theta\,dz = 4\pi\left(1-\frac{1}{e^4}\right)$

16.5 Homework

(5) $F = \langle y, z, x \rangle$; plane $3x - 4y + z = 1$; upward-pointing normal $0 \le x \le 1$, $0 \le y \le 1$

$\rightarrow \iint_D \left(-P \frac{dg}{dx} - Q \frac{dg}{dy} + R\right) dA$

$\rightarrow g(x,y) = -3x + 1 + 4y$; $g_x = -3$, $g_y = 4$

$\rightarrow \int_0^1 \int_0^1 (13x - 13y - 4)\, dx\, dy = -4$

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(7) $F = \langle 0, 3, x \rangle$, part of sphere $x^2 + y^2 + z^2 = 9$, $x \ge 0$, $y \ge 0$, $z \ge 0$, outward

$\rightarrow \int_0^{\pi/2} \int_0^{\pi/2} \left(27 \sin\theta \sin^2\phi + 27 \cos\theta \sin^2\phi \cos\theta\right) d\theta\, d\phi = \frac{27}{12}(3\pi + 4)$

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(9) Not too sure so I will look into this.

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(11) $F = y^2 i + 2j + xk$, $x + y + z = 1$

$\rightarrow \int_0^1 \int_0^{\pi/2} \left(r^2 \sin^2\theta + 2 + r\cos\theta\right) d\theta\, dr = \frac{11}{12}$
