HW 16.4- 16.5 due 11/22/20

16.4

7. $G(u,v) = (2u+v, u-4v, 3u)$
$T_u = \langle 2, 1, 3 \rangle \quad T_v = \langle 1, -4, 0 \rangle$
$N = T_u \times T_v = \langle 12, 3, -9 \rangle$
For $u=1, v=4$, $12(x-6) + 3(y+15) - 9(z-3) = 0$
$\boxed{4x + y - 3z = 0}$

13. $G(u,v) = (u\cos v, u\sin v, u)$ $f(x,y,z) = z\sqrt{x^2+y^2}$
$x = u\cos v \quad y = u\sin v \quad z = u$
$\int_0^1\int_0^1 u(u^2\cos^2 v + u^2\sin^2 v)\,ds$
$\int_0^1\int_0^1 u^3\,ds = \int_0^1 \frac{1}{5}\,dv = \frac{\sqrt{2}}{5}$

15. $y = 9 - z^2$, $0 \le x \le 3$, $0 \le z \le 3$, $f(x,y,z) = z$
$y = g(y,z)$ $dS = \sqrt{1+4z^2}$
$\int_0^3\int_0^3 z\sqrt{1+4z^2}\,dx\,dz \rightarrow 3\int_0^3 z\sqrt{1+4z^2}\,dz$ u-sub $= 56.02$

19. $x^2+y^2 = 4 \quad 0 \le z \le 4 \quad f(x,y,z) = e^{-z}$
$\langle 2\cos\theta, 2\sin\theta, z \rangle$
$\| \langle -2\sin\theta, 2\cos\theta, 0 \rangle \times \langle 0, 0, 1 \rangle \| = 2 \quad dS = 2$
$2\int_0^{2\pi}\int_0^4 e^{-t}\,dr\,d\theta = 4\pi(1-e^{-4})$

16.5

5. $F = \langle y, z, x \rangle$ plane $3x - 4y + z = 1$ $0 \le x \le 1$, $0 \le y \le 1$
$\iint_D 3y - 4 + 12x - 16y + x\,dy\,dx$
$\int \frac{13}{2} - 13y - 4 \rightarrow \frac{13}{2}y - \frac{13}{2}y^2 - 4y = -4$

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7. $F = \langle 0, 3, x \rangle$ part of sphere $x^2 + y^2 + z^2 = 9$ $x \geq 0$ $y \geq 0$ $z \geq 0$

$\vec{r}(\theta, \phi) = 3\sin\phi\cos\theta\, i + 3\sin\phi\sin\theta\, j + 3\cos\phi\, k$

$0 \leq \theta \leq \frac{\pi}{2}$ $\quad 0 \leq \phi \leq \frac{\pi}{2}$

$r_\theta = -3\sin\phi\sin\theta + 3\sin\phi\cos\theta j$

$r_\phi = 3\cos\phi\cos\theta\, i + 3\cos\phi\sin\theta\, j - 3\sin\phi\, k$

$r_\theta \times r_\phi = -9\sin^2\phi\cos\theta\, i - 9\sin^2\phi\sin\theta\, j - 9\sin\phi\cos\phi\, k$

$n = \frac{r_\theta \times r_\phi}{\|r_\theta \times r_\phi\|} \rightarrow \iint F \cdot dS = \iint F \cdot \left(\frac{r_\theta \times r_\phi}{\|r_\theta \times r_\phi\|}\right) \|\vec{r}_\theta \times \vec{r}_\phi\|\, dA$

* Not sure where to go from here

9. $F = \langle z, z, x \rangle$ $z = 9 - x^2 - y^2$ $x \geq 0$ $y \leq 2$ $y \geq 0$ $z \geq 0$

$\int_0^3 \int_0^2 18x - 2x^3 - 2xy^2 + 18y - 2x^2y - 2y^3 + x\, dy\, dx$

$\int_0^3 54x - 6x^3 - 18x + 18 - 9x^2 - \frac{81}{4} + 3x\, dx$

$27x^2 - \frac{3}{2}x^4 - 9x^2 + 81x - 3x^3 - \frac{81}{4}x + \frac{3}{2}x^2 \Big|_0^3$

$1693/5$

11. $F = \langle y^2, 2, -x \rangle$ $\quad x + y + z = 1$ $\quad x, y, z \geq 0$

$\iint -y^2(-1) + 2 - x\, dy\, dx$

$y^2 + 2 - x\, dy\, dx$

$\frac{y^3}{3} + 2y - yx \Big|_0^1 = \int_0^1 \frac{1}{3} + 2 - x\, dx$

$\frac{1}{3} + 2 - \frac{x^2}{2} \Big|_0 = \left(\frac{1}{3} + 2 - \frac{1}{2}\right)\frac{1}{2} = 11/12$