16.4 Homework

7. $G(u,v) = (2u+v,\ u-4v,\ 3u)$

$T_u = \langle 2, 1, 3\rangle \qquad T_v = \langle 1, -4, 0\rangle$

$N(u,v) = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & 1 & 3 \\ 1 & -4 & 0 \end{vmatrix} = \hat{i}(0+12) - \hat{j}(0-3) + \hat{k}(-8-1)$

$12\hat{i} + 3\hat{j} - 9\hat{k}$

$\langle 12, 3, -9\rangle$ At $(1,4)$

$\Rightarrow (6, -15, 3)$

$12(x-6) + 3(y+15) - 9(z-3)$

$12x - 72 + 3y + 45 - 9z + 27 = 0$

13. $G(u,v) = (u\cos v,\ u\sin v,\ u)$, $0 \le u \le 1$ $\quad 0 \le v \le 1$

$f(x,y,z) = z(x^2+y^2)$

$\iint_S z(x^2+y^2)\,dS$

$r = \langle u\cos v, u\sin v, u\rangle$

$r_u = \langle \cos v, \sin v, 1\rangle$

$r_v = \langle -u\sin v, u\cos v, 0\rangle$

$\|r_u \times r_v\| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos v & \sin v & 1 \\ -u\sin v & u\cos v & 0 \end{vmatrix}$ $\quad \hat{i}(-u\cos v) - \hat{j}(u\sin v) + \hat{k}(u\cos^2 v + u\sin^2 v)$

$\hat{i}(-u\cos v) - \hat{j}(u\sin v) + \hat{k}(u)$

$\langle -u\cos v, -u\sin v, u\rangle$

$\sqrt{u^2\cos^2 v + u^2\sin^2 v + u^2} = \sqrt{u^2(2)} = u\sqrt{2}$

$dS = 2\sqrt{u}\,du\,dv$

$\int_0^1\int_0^1 u(u^2\cos^2 v + u^2\sin^2 v)\,du\,dv = \int_0^1\int_0^1 u(u^2(1))\,du\,dv$

$= \int_0^1\int_0^1 2u^3 \cdot u^{1/2}\,du\,dv = \sqrt{2}\int_0^1\int_0^1 u^4\,du\,dv = \frac{\sqrt{2}}{5}$

15. $y = 9 - z^2 \quad 0 \le x \le 3 \quad 0 \le z \le 3 \quad f(x,y,z) = z$

$$\iint_S z \, dS$$

$x = x \quad y = 9 - z^2 \quad z = z$

$r = \langle u, 9 - v^2, v \rangle$

$r_u = \langle 1, 0, 0 \rangle$

$r_v = \langle 0, -2v, 1 \rangle$

$$r_u \times r_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 0 & -2v & 1 \end{vmatrix} = \begin{matrix} \hat{i}(0) - \hat{j}(1) + \hat{k}(-2v) \\ -\hat{j} + \hat{k}(-2v) \\ \langle 0, -1, -2v \rangle \end{matrix}$$

$|r_u \times r_v| = \sqrt{1 + 4v^2}$

$dS = \sqrt{1+4v^2} \, du \, dv$

$$\int_0^3 \int_0^3 v \cdot \sqrt{1+4v^2} \, du \, dv = \frac{37\sqrt{37}}{4} - \frac{1}{4} \approx 56.02$$

19. $x^2 + y^2 = 4 \quad 0 \le z \le 4 \quad f(x,y,z) = e^{-z}$

$$\iint_S e^{-z} \, dS$$

Circle with $R = 2$

$r = \langle 2\cos\theta, 2\sin\theta, r \rangle \quad 0 \le \theta \le 2\pi \quad 0 \le r \le 2$

$r_r = \langle 0, 0, 1 \rangle$

$r_\theta = \langle -2\sin\theta, 2\cos\theta, 0 \rangle$

$$r_r \times r_\theta = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 0 & 0 & 1 \\ -2\sin\theta & 2\cos\theta & 0 \end{vmatrix} = \begin{matrix} \hat{i}(0 - 2\cos\theta) - \hat{j}(2\sin\theta) + \hat{k}(0) \\ \langle -2\cos\theta, -2\sin\theta, 0 \rangle \end{matrix}$$

$|r_r \times r_\theta| = \sqrt{4\cos^2\theta + 4\sin^2\theta} = \sqrt{4} = 2$

$dS = 2 \, dr \, d\theta$

$$\int_0^{2\pi} \int_0^2 2e^{-r} \, dr \, d\theta = 4\pi - 4e^{-2}\pi$$

16.5 Homework

5. $F = \langle y, z, x \rangle$ $\quad 3x - 4y + z = 1$

$z = 1 - 3x + 4y$

$0 \le x \le 1 \qquad 0 \le y \le 1$

$g = 1 - 3x + 4y \qquad P = y \qquad Q = z \qquad R = x$

$\iint_S F \cdot dS =$

$\frac{\partial g}{\partial x} = -3 \qquad \frac{\partial g}{\partial y} = 4$

$\iint_S \left(-y \cdot -3 - z \cdot 4 + x\right) dA$

$= \iint_S (3y - 4z + x)\, dA$ Replace z

$= \iint_S (3y - (1 - 3x + 4y) + x)\, dA$

$= \iint_S (3y - 1 + 3x + 4y + x) = \iint_S (4x + 7y - 1)\, dA$

$= \int_0^1 \int_0^1 (4x + 7y - 1)\, dy\, dx = \frac{9}{2}$

7. $F = \langle y, z, x \rangle$ $\quad x^2 + y^2 + z^2 = 9 \qquad z = \sqrt{9 - x^2 - y^2}$

$P = y \qquad Q = z \qquad R = x$

$g = \sqrt{9 - x^2 - y^2}$

$\frac{\partial g}{\partial x} = \frac{-x}{\sqrt{-x^2 - y^2 + 9}} = \frac{-x}{\sqrt{9 - x^2 - y^2}} \qquad \frac{\partial g}{\partial y} = \frac{-y}{\sqrt{9 - x^2 - y^2}}$

$\frac{xy}{\sqrt{9 - x^2 - y^2}} + \frac{y \cdot \sqrt{9 - x^2 - y^2}}{\sqrt{9 - x^2 - y^2}} + x = \frac{xy}{\sqrt{9 - x^2 - y^2}} + y + x \; dA$

$0 \le r \le 3$

$0 \le \theta \le \pi/2$

9. $F = \langle z, z, x \rangle$ $\quad z = 9 - x^2 - y^2 \quad x \geq 0 \quad y \geq 0 \quad z \geq 0$

$P = z \quad Q = z \quad R = x$

$g = 9 - x^2 - y^2$

$\frac{\partial g}{\partial x} = -2x \quad \frac{\partial g}{\partial y} = -2y$

$(-9 + x^2 + y^2)(-2x) + (-9 + x^2 + y^2)(-2y) + x$

$$\int_0^{\pi/2}\int_0^3 (-9 + r^2)(-2r\cos(\theta)) + (-9 + r^2)(-2\sin(\theta)) + r\cos\theta \, dr\, d\theta$$

$= 85.5$

11. $\langle y^2, 2, -x \rangle$ $\quad P = y^2 \quad Q = 2 \quad R = -x$

$z = 1 - x - y$ $\quad g = 1 - x - y$

$\frac{\partial g}{\partial x} = -1 \quad \frac{\partial g}{\partial y} = -1$

$\iint (-y^2 \cdot -1) - (2 \cdot -1) - x$

$x + y + z = 0$,1

$$\int_0^1\int_0^{1-y} y^2 + 2 - x \, \cancel{dA}\, dx = \frac{11}{12}$$