So path integrals are also equal

16.4/16.5 Homework

16.4

7. $T_u = \langle 2, 1, 3 \rangle$
$T_v = \langle 1, -4, 0 \rangle$

$N = T_u \times T_v = \langle 12, 3, -9 \rangle$
$\sigma(1,4) = (6, -15, 3)$

$12(x-6) + 3(y+15) - 9(z-3) = 0$
$12x - 72 + 3y + 45 - 9z + 27 = 0$
$4x - 24 + y + 15 - 3z + 9 = 0$
$\boxed{4x + y - 3z = 0}$

13.
$x = u\cos v;\ 0 \le u \le 1$
$y = u\sin v;\ 0 \le v \le 1$
$z = u$

$dS = |\langle \cos v, \sin v, 1 \rangle \times \langle -us$ ... $dudv$
$dS = u\sqrt{2}\, dudv$

$\iint_S u(u^2(\sin^2 v + \cos^2 v))\, dS$

$= \iint_S u^3\, dS$

$\sqrt{2} \int_0^1 \int_0^1 u^4\, du\, dv = \sqrt{2} \int_0^1 \frac{1}{5}\, dv$ $\boxed{= \frac{\sqrt{2}}{5}}$

15. $\iint_S z\, dS$ $y = g(x,z)$ $dS = \sqrt{1+4z^2}$

$\int_0^3\int_0^3 z\sqrt{1+4z^2}\,dx\,dz = 3\int_0^3 z\sqrt{1+4z^2}\,dz$

$u = 1+4z^2$
$du = 8z\,dx$

$= \frac{3}{8}\int_1^{37} u^{1/2}\,du = \frac{3}{8}\left(\frac{2}{3}u^{3/2}\right)\Big|_1^{37}$

$\frac{1}{4}[37^{3/2} - 1]$ $\boxed{= 56.02}$

19. Cartesian → Cylindrical
$x^2+y^2 = 4$

$0 \le \theta \le 2\pi$
$0 \le r \le 4$ $z = r$

$\langle 2\cos\theta, 2\sin\theta, z\rangle$
$|\langle -2\sin\theta, 2\cos\theta, 0\rangle \times \langle 0,0,1\rangle|$
$= |\langle 2\cos\theta, 2\sin\theta, 0\rangle| = 2$ $dS = 2$

$2\int_0^{2\pi}\int_0^4 e^{-r}\,dr\,d\theta$ $\boxed{= 4\pi(1 - e^{-4})}$

16.5 $z = 1-3x+4y$

5. $\iint_D -y(-3) - z(4) + x\, dA$

$= \int_0^1\int_0^1 3y - 4 + 12x - 16y + x\, dy\,dx$

$= \int_0^1\int_0^1 13x - 13y - 4\, dy\,dx$

$= \int_0^1 \frac{13}{2} - 13y - 4 = \frac{13}{2}y - \frac{13}{2}y^2 - 4y$

$= \frac{13}{2} - \frac{13}{2} - 4 = \boxed{-4}$

7. $\iint_S F \cdot dS$ Cartesian → Spherical

$F = \langle 0, 3, 3\sin\varphi\cos\varphi \rangle$ $r = \langle 3\sin\varphi\cos\theta, 3\sin\varphi\sin\theta, 3\cos\varphi \rangle$

$dS = |r_\theta \times r_\varphi| \, d\theta \, d\varphi$ $r_\theta = \langle -3\sin\varphi\sin\theta, 3\sin\varphi\cos\theta, 0 \rangle$

$r_\varphi = \langle 3\cos\varphi\cos\theta, 3\cos\varphi\sin\theta, -3\sin\varphi \rangle$

$|r_\theta \times r_\varphi| = \sqrt{(9\sin\varphi^2\cos\theta)^2 + (9\sin\varphi^2\sin\theta)^2 + (-9\sin\varphi\cos\varphi)}$

$=$

I'm not sure how to go past this point, Sorry Dr. Z

9.

$F = \langle 9-x^2-y^2, 9-x^2-y^2, x \rangle$

$r_x = \langle 1, 0, -2x$ $r_x \times r_y = \langle 2x, 2y, 1 \rangle$

$r_y = \langle 0, 1, -2y \rangle$

$\int_0^3\int_0^3 F \cdot (r_x \times r_y) = \int_0^3\int_0^3 -2x^3 - 2x^2y - 2xy^2 + 18x - 2y^3 + 18y \, dy\,dx$

= USED MAPLE TO EVALUATE → $\boxed{\frac{189}{2}}$

11. $\int_0^1\int_0^1 -(y^2)(-1) - 2(-1) - x$

$z = 1-x-y$
$0 \le x \le 1$
$0 \le y \le 1$

$\int_0^1\int_0^1 y^2 + 2 - x \, dy \, dx$

$\int_0^1 \frac{y^3}{3} + 2y - xy \Big|_0^1 dx$

$= \int_0^1 \frac{1}{3} + 2 - x \, dx = \frac{1}{3}x + 2x - \frac{x^2}{2}\Big|_0^1$

$= \frac{1}{3} + 2 - \frac{1}{2} = \boxed{\frac{11}{6}}$