$16.4 - 7, 13, 15, 19$
$16.5 - 5, 7, 9, 11$

$\boxed{16.4,\ 16.5}$

At $u = 1$, $v = 4$

$x = 2u + v$ — 6
$y = u - 4v$ — -15
$z = 3u$ — 3

$1 - 4(4) = 1 - 16$
$z = 3(1)$

⑦ $G(u,v) = (2u+v,\ u-4v,\ 3u)$ $\quad u=1,\ v=4$

$r = (2u+v)\hat{i} + (u-4v)\hat{j} + 3u\hat{k}$
$r_u = 2\hat{i} + \hat{j} + 3\hat{k} = \langle 2, 1, 3\rangle$
$r_v = \hat{u} - 4\hat{j} + 0\hat{k} = \langle 1, -4, 0\rangle$

$(r_u \times r_v)$

$$\begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ 1 & -4 & 0 \end{vmatrix}$$

$i(12) - j(-3) + k(-9)$

$= 12\hat{i} + 3\hat{j} - 9\hat{k} = \langle 12, 3, -9\rangle$

$= \langle \cancel{3}\,4, 1, -3\rangle$

~~Equation~~ — $(x, y, z) = (6, -15, 3)$

Equation: $4(x-6) + 1(y+15) - 3(z-3)$

$0 = 4x - 24 + y + 15 - 3z + 9$

$= \boxed{4x + y - 3z - 48 = 0}$

$15 + 9 = 24$

⑬ $G(u,v) = (u\cos v,\ u\sin v,\ u)$ $\quad \boxed{0 \le u \le 1},\ \boxed{0 \le v \le 1}$

$f(x, y, z) = z(x^2 + y^2)$

$$\boxed{\begin{aligned} x &= u\cos v \\ y &= u\sin v \\ z &= u \end{aligned}}$$

$u(u^2\cos^2 v + u^2\sin^2 v)$

$= u(u^2(1))$

$\boxed{f(u) = u^3}$

Page No.
Date

$\cancel{G(v,v)} = \cancel{t}$

$\iint_S f(x,y,z)\,ds$

$$ds = \sqrt{1 + \left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}\right)^2}$$

$$ds = |r_u \times r_v|\,du\,dv$$

$ds =$

$r = \langle u\cos v, u\sin v, u\rangle$

$r = u\cos v\,\hat{i} + u\sin v\,\hat{j} + u\hat{k}$

$r_u = \langle \cos v, \sin v, 1\rangle$

$r_v = \langle -u\sin v, u\cos v, 0\rangle$

$$\int_0^1\int_0^1 u^3$$

$$\begin{vmatrix} i & j & k \\ \cos v & \sin v & 1 \\ -u\sin v & u\cos v & 0 \end{vmatrix}$$

$= (-u\cos v)\hat{i} - u\sin v\,\hat{j} + u$ $\quad (-u\cos v)\hat{i} - (u\sin v)\hat{j} +$

$-u(\cos v + \sin j)$ $\quad (u\cos^2 v + u\sin^2 v)$

$\langle$

$\cancel{u\cos\theta\,\hat{i}}$

$= (-u\cos v - u \cancel{\cos}\sin v + u)$

$\sqrt{u^2\cos^2 v + u^2\sin^2 v + u^2}$

$= \sqrt{u^2(\sin^2 v + \cos^2 v) + u^2}$

$= \sqrt{u^2(1) + u^2}$

$= \sqrt{2u^2}$

$= \cancel{\boxed{u^2}} \quad \sqrt{2}u.$

$\int_0^1 \int_0^1 u^3 \cdot v^2 \, du\,dv$

$\int_0^1 \int_0^1 u^5$

$\frac{u^6}{6}\Big|_0^1$

$= \int_0^1 \frac{1}{6}\, dv$

$= \frac{1}{6} v \Big|_0^1$

$= \frac{1}{6}$

$\int_0^1 \int_0^1 \sqrt{2u} \cdot u^4$

$\int_0^1 \sqrt{2}\, \frac{u^5}{5}\Big|_0^1$

$\int_0^1 \frac{\sqrt{2}}{5}\, dv$

$= \frac{\sqrt{2}}{5}$

Page No.
Date

3) $y = 9 - z^2$ $\quad 0 \le x \le 3,\ 0 \le z \le 3$ $\quad \boxed{f(x,y,z) = z}$

$\boxed{0 \le y \le 9}$

$z^2 = 9 - y$

$\boxed{z = \sqrt{9-y}}$

$$\left(\frac{-1}{2\sqrt{9-y}}\right)^2$$

$$\iint_S f(x,y,z)\, ds$$

$$= \frac{1}{4(9-y)}$$

$$= \frac{1}{36-4y}$$

$$ds = \sqrt{1 + \left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}\right)^2}$$

$$= \sqrt{1 + (0)^2 + \left(\frac{1}{36-4y}\right)}\, dA$$

$$= \sqrt{\frac{36-4y+1}{36-4y}}$$

$$= \sqrt{\frac{37-4y}{36-4y}}\, dA$$

$$\int_0^9 \int_0^3 \left(\sqrt{9-y}\right)\left(\sqrt{\frac{37-4y}{36-4y}}\right) dA$$

$$= \frac{37\sqrt{37}-1}{4}$$

$4\pi(1-e^{-4}) = 12.33$

(19) $x^2+y^2=4 \quad 0 \le z \le 4 \quad f(x,y,z) = e^{-z}$

$x = r\cos\theta$

$y = r\sin\theta$

$z = r$

$$\iint_S f(x,y,z)\,ds = \int_0^2\int_0^2 e^{-(x^2+y^2)}\sqrt{2}\,dx\,dy$$

$$z^2 = x^2+y^2$$

$$z = \sqrt{x^2+y^2}$$

$$= \frac{1}{4}\pi$$

e^{-r}

$$ds = \frac{dz}{dx} = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2}}$$

$$\frac{dz}{dy} = \frac{\cancel{2}y}{\cancel{2}\sqrt{x^2+y^2}}$$

$r = r\cos\theta\,\hat{i} \times r\sin\theta\,\hat{j} \times r\hat{k}$

$r_r = \cos\theta\,\hat{i} \times \sin\theta\,\hat{j} \times \hat{k}$

$r_\theta = -r\sin\theta\,\hat{i}\; r\cos\theta\,\hat{j}\; 0\hat{k}$

$(-r\cos\theta,\; -r\sin\theta,\; r)$

$$\sqrt{1 + \left(\frac{x^2}{x^2+y^2}\right) + \left(\frac{y^2}{x^2+y^2}\right)}$$

$$= \sqrt{1 + \frac{x^2+y^2}{x^2+y^2}}$$

$$\sqrt{1+1}$$

$$= \sqrt{2}$$

$r^2\cos^2\theta \quad r^2\sin^2\theta \quad r^2 \quad r^2$

$r^2(1)$

r^2

$\sqrt{2}\,r$

$\boxed{16.5}$

3, 7, 9, 11 $\iint_S F \cdot dS$

3. $F = \langle y, z, x \rangle$, plane $= 3x - 4y + z = 1$

$0 \le x \le 1$, $0 \le y \le 1$, upward pointing normal.

$$\boxed{\begin{aligned} P &= y \\ Q &= z \\ R &= x \end{aligned}}$$

$$\boxed{z = 1 + 4y - 3x}$$

$$\iint_{D_1} \left(-P \frac{dz}{dx} - Q \frac{dz}{dy} + R \right) dA$$

$$\int_0^1 \int_0^1 \left(-y(-3) - z(4) + x \right) dx\, dy$$

$$\int_0^1 \int_0^1 (3y - 4z + x)\, dx\, dy$$

$$(3y - 4(1 + 4y - 3x) + x)$$

$$(3y - 4 - 16y + 12x + x)$$

$$\iint (-4 - 13y + 13x) = -4$$