Exercise 16.2

Q3. $F = (y^2, x^2)$. $C = y = x^{-1}$ $1 \le x \le 2$ oriented from left to right.

(a) $F(r(t))$ & $dr = r'(t)dt$

$\because r(t) \Rightarrow$ ~~$x = r\cos t$ $y = r\sin t$~~ $r(t) = (t, t^{-1})$

~~$r = \sqrt{x^2+y^2} = \sqrt{x^2+x^{-2}}$~~ $= F(r(t)) = (t^{-2}, t^2)$

$r'(t) = (1, -t^{-2})$

(b) $F(r(t)) \cdot r'(t)dt$

$= (t^{-2}, t^2) \cdot (1, -t^{-2})\,dt$

$= (t^{-2} - 1)dt$

$\int_C F \cdot dr$

$= \int_1^2 (t^{-1} - 1)\,dt$

$= -\frac{1}{2}$

KOKUYO

Q9. $f(x,y) = \sqrt{1+9xy}$ $\quad y = x^3$ for $0 \le x \le 1$

$\because (x, x^3) \quad 0 \le x \le 1$

$= (t, t^3) \quad 0 \le t \le 1$

$\therefore \ ds = \sqrt{1+(3t^2)^2}$

$\int_0^1 \sqrt{1+9t \cdot t^3} \cdot \sqrt{1+9t^4} \ dt$

$= \int_0^1 (1+9t^4) \ dt$

$= 2.8$

Q11. $f(x,y,z)=z^2$, $r(t)=(2t,3t,4t)$ for $0\le t\le 2$

$f(r(t))=16t^2$

$r'(t)=(2,3,4)$

$\therefore \int_0^2 16t^2\cdot \cancel{\sqrt{2^2+3^2+4^2}}\,dt$ $\quad \sqrt{2^2+3^2+4^2}$

$=\int_0^2 16t^2\cdot\sqrt{29}\,dt$

Campus

$=229.767\approx 229.8$

(0,2,0) to (1,1,1) ✓

分段线性路径.

Q13. $f(x,y,z)=xe^{z^2}$, piecewise linear path from (0,0,1) to

let point A be (0,0,1) B be (0,2,0) C be (1,1,1)

AB: $A+t\cdot(B-A)=(0,0,1)+t(0,2,-1)$

$=(0,2t,1-t)$

$ds=\sqrt{4+1}$ $\quad x=0,\ y=2t,\ z=1-t$

$=\sqrt{5}$ $\quad \cancel{ds=\sqrt{0^2+4t^2+(1-t)^2}\,dt=\sqrt{4t^2+t^2+1-2t}}$

$\cancel{=\sqrt{5t^2+1-2t}}$

$\therefore \int_0^1 \cancel{\sqrt{5t^2+1-2t}}\cdot 0\cdot e^{(1-t)^2}\,dt=0.$ $\quad \sqrt{5}$

BC: $B+t(C-B)=(0,2,0)+t(1,-1,1)$

$=(t,2-t,t)$

$x=t\quad y=2-t\quad z=t$

$ds=\cancel{\sqrt{t^2+(2-t)^2+t^2}=\sqrt{3t^2-4t+4}}\ \sqrt{1+1+1}=\sqrt{3}$

$\int_0^1 t\cdot e^{t^2}\cdot\cancel{\sqrt{3t^2-4t+4}}\,dt$ $\quad \sqrt{3}$

$=\cancel{1.4429}\ 1.488$

CA $=C+t(A-C)$ $\quad x=1-t,\ y=1-t$

$=(1,1,1)+t(-1,-1,0)$ $\quad z=1$

$=(1-t,1-t,1)$

$ds=\cancel{\sqrt{(1-t)^2+(1-t)^2+1^2}=\sqrt{3+2t^2-4t}}$ $\quad \sqrt{1+1+1}=\sqrt{3}$

$\int_0^1 (1-t)\cdot e^1\cdot\cancel{\sqrt{3+2t^2-4t}}\,dt.=\cancel{5.0103}$ $\quad 7.0623$

$\sqrt{3}$

No.

Date

Q17. $\int_C 1\,ds = ?$ $\quad r(t) = (4t, -3t, 12t) \quad 2 \le t \le 5$

$r'(t) = (4, -3, 12)$

$= \sqrt{4^2 + (-3)^2 + 12^2} = 13$

$\int_2^5 1 \cdot 13\,dt = 39$

the integral means the distance from (8, -6, 24) and (20, -15, 60) at plane is equal to 1.

Q27. $\int_C y\,dx - x\,dy \quad y = x^2 \quad 0 \le x \le 2$

~~$x = r\cos t = 0, 2$~~

~~$r\cos t = 0$ or 2~~

~~$y = r\sin t = (r\cos t)^2$~~

~~$\therefore r = \sqrt{(r\cos t)^2 + (r\cos t)^4}$~~

~~$= \sqrt{0}$ or $\sqrt{4+16}$~~

~~$= 0$ or $\sqrt{20}$~~

$\therefore x = \sqrt{20}\cos t \quad y = \sqrt{20}\sin t$

$\therefore 0 = \sqrt{20}\cos t, \quad t = \frac{1}{2}\pi$

~~$2 = \sqrt{20}\cos t, \quad t =$~~

$y = x^2 \quad 0 \le x \le 2 \quad \therefore (x, x^2) \quad 0 \le x \le 2$

$(t, t^2) \quad 0 \le t \le 2.$

$x(t) = t \quad y(t) = t^2 \qquad dx = 1\,dt \quad dy = 2t\,dt.$

$ds = \sqrt{1 + 4t^2} = \sqrt{1}$ or $\sqrt{17} = 1$ or $\sqrt{17}.$

$\int_0^2 t^2 \cdot dt - t \cdot 2t\,dt$

$= \int_0^2 t^2 - 2t^2\,dt = \int_0^2 -t^2\,dt$

$= -\frac{1}{3}t^3 \Big|_0^2$

$= -\frac{8}{3}$

Campus

No.

Date

Q29. $\int_C (x-y)\,dx+(y-z)\,dy+z\,dz$ $(0,0,0)$ to $(1,4,4)$

$(0,0,0)+t\cdot(1,4,4)=(1t,4t,4t)$

$ds=\sqrt{1+16+16}=\sqrt{33}$

$\int_0^1 (t-4t)\cdot dt+(4t-4t)4\,dt+4t\cdot dt\cdot 4$

$=\int_0^1 -3t+16t\,dt=\int_0^1 13t\,dt$

$=\frac{13}{2}$

Q31. $\int_C \frac{-y\,dx+x\,dy}{x^2+y^2}$ $(1,0)$ to $(0,1)$

$(1,0)+t\cdot(-1,1)=(1-t,t)$

$x(t)=(1-t),\quad y(t)=t$

$dx=-1\,dt,\quad dy=1\,dt$

$\int_0^1 \frac{-t\cdot -1\,dt+(1-t)\,dt}{(1-t)^2+t^2}$

$=\int_0^1 \frac{t+1-t}{1-2t+2t^2}\,dt$

$=\frac{\pi}{2}\approx 1.5708$

Q35. $F(x,y,z)=(e^z,e^{x-y},e^y)$

$P=(0,0,0)$ $A=(0,0,1)$ $B=(0,1,1)$ $Q=(-1,1,1)$

PA: $(0,0,0)+(0,0,1)t=(0,0,t)$ ~~$ds\sqrt{1^2}=1$~~

$F(x,y,z)=(e^t,1,1)$

$\int_0^1 e^t\cdot 0\,dt+1\cdot 0\,dt+1\cdot 1\,dt$

$=\int_0^1 1\,dt=1$

KOKUYO

No.

Date

AB: $(0,0,1)+t(0,1,-1)=(0,t,1-t)$

$F(x,y,z)=(e^{1-t}, e^{-t}, e^{t})$

$\int_0^1 e^{1-t}\cdot 0\,dt + e^{-t}\cdot 1\,dt + e^{t}\cdot(-1)\,dt$

$=\int_0^1 e^{-t}-e^{t}\,dt$

$=-1.086$

BQ: $(0,1,1)+t(-1,0,0)=(-t,1,1)$

$F(x,y,z)=(e^{1}, e^{-t-1}, e^{1})$

$\int_0^1 e^{1}(-1)\,dt + e^{-t-1}\cdot 0\,dt + 0$

$=\int_0^1 -e^{1}\,dt$

$=-2.718$

Exercise 16.3

Q1. let $f(x,y,z) = xy\sin(yz)$, $F = \nabla f$, evaluate $\int_C F\,dr$ where c is any path from $(0,0,0)$ to $(1,1,\pi)$

$\because f = xy\sin(yz)$

$F = \nabla f = (y\sin(yz),\ y\sin(yz) + xy\cdot\cos(yz)\cdot z,\ xy^2\cos(yz))$

$\because F = \nabla f$

$\therefore \int_C F\,dr = f(Q) - f(P)$

$= f(1,1,\pi) - f(0,0,0)$

$= 1\cdot\sin\pi - 0 = 0$

Q3. $F(x,y) = (3, 6y)$, $f(x,y) = 3x + 3y^2$, $r(t) = (t, 2t^{-1})$ for $1 \le t \le 4$

verify $F = \nabla f$ & evaluate the line integral of F.

$\because F(x,y) = \nabla f = (3, 6y)$

$\therefore (3, 6y) = (3, 6y)$

$r(t) = (t, 2t^{-1})$

$r'(t) = (1, -2t^{-2})$

$\int_C F\,dr = \int_1^4 (3, 6\cdot 2t^{-1}).(1, -2t^{-2})\,dt$

$= \int_1^4 3 - 24t^{-3}\,dt$

$= 3t + 6t^{-4}\Big|_1^4$

$= -\frac{9}{4}$

Q5. $F(x,y,z) = ye^z i + xe^z j + xye^z k$ $\quad f(x,y,z) = xye^z$;

$r(t) = (t^2, t^3, t-1)$ for $1 \le t \le 2$.

$\because F = \nabla f = (ye^z, xe^z, xye^z)$

it is conservative.

$\because r(t) = (t^2, t^3, t-1)$

$r'(t) = (2t, 3t^2, 1)$

$\int_C F\,dr = \int_1^2 (ye^z, xe^z, xye^z).(2t, 3t^2, 1)\,dt$

$= \int_1^2 (t^3e^{t-1}, t^2\cdot e^{t-1}, t^2\cdot t^3 e^{t-1}).(2t, 3t^2, 1)\,dt$

$= \int_1^2 (2t^4e^{t-1} + 3t^4e^{t-1} + t^5e^{t-1})\,dt$

$= \int_1^2 (5t^4 e^{t-1} + t^5 e^{t-1}) dt$

$= t^5 e^{t-1} \Big|_1^2$

$= 2^5 \cdot e^1 - 1 \cdot e^0$

$= 32e - 1$

$= 85.985$

Q9. $F = y^2 i + (2xy + e^z) j + ye^z k$
find potential function for F or determine that F is not conser.

Answer: $\because F = y^2 i + (2xy + e^z) j + ye^z k$

$$\text{curl } F = \nabla \times F = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ y^2 & 2xy+e^z & ye^z \end{vmatrix}$$

$= i(e^z - e^z) - j(0-0) + k(2y - 2y)$

$= (0,0,0)$

$\therefore$ F is conservative.

$F = \nabla f = (f_x, f_y, f_z)$

$f_x = y^2$

$f = \int y^2 dx = xy^2 + g(y,z)$

$\frac{d}{dy} f = 2yx + g_y(y,z) = 2xy + e^z$

$\therefore g_y(y,z) = e^z$ ~~$g(y,z) = e^z + h(z)$~~

$g(y,z) = e^z y + h(z)$

$f = xy^2 + e^z y + h(z)$

$\frac{d}{dz} f = e^z y + h'(z) = ye^z$

$h'(z) = 0$

$h(z) = 0$

$\therefore f = e^z \cdot y + xy^2$

Q13. $F = (z\sec^2 x, z, y + \tan x)$.

$$\text{curl} F = \nabla F = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ z\sec^2 x & z & y+\tan x \end{vmatrix}$$

$= (1-1)i - j(\cancel{0}\sec^2 x - \sec^2 x) + (0-0)k$

$= (0,0,0)$.

$\therefore F$ is conservative.

$F = \nabla f \Rightarrow (z\sec^2 x, z, y+\tan x)$

$f_x = z\sec^2 x$

$f = \int f_x \, dx = z\tan x + g(y,z)$.

$\frac{d}{dy}(f) = g'_y(y,z) = z$

$g(y,z) = zy + h(z)$

$f = z\tan x + zy + h(z)$

$\frac{d}{dz} = \tan x + y + h'(z) = y + \tan x$

$h'(z) = 0$

$h(z) = 0$

$\therefore f = z\tan x + zy$

Q15. $F = (2xy+5, x^2-4z, -4y)$

$$\text{curl} F = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 2xy+5 & x^2-4z & -4y \end{vmatrix} = (-4+4)i - j(0-0) + k(2x-2x)$$

$= (0,0,0)$

it is conservative.

$F = \nabla f = (2xy+5, x^2-4z, -4y)$

$f = \int f_x \, dx = x^2 y + 5x + g(y,z)$

$\frac{d}{dy} f = x^2 + g_y(y,z) = x^2 - 4z$

$\therefore g_y(y,z) = -4z \quad g(y,z) = -4yz + h(z)$ ✓

$f = x^2y + 5x - 4yz + h(z)$

$\frac{d}{dz} = -4y + h'(z) = -4y$

$\therefore h'(z) = 0 \quad h(z) = 0$

$\therefore f = x^2y + 5x - 4yz$

Q17. $\int_C 2xyz\,dx + x^2z\,dy + x^2y\,dz$

$r(t) = (t^2, \sin\frac{\pi t}{4}, e^{t^2-2t})$ for $0 \leq t \leq 2$.

A: $\text{curl } F = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 2xyz & x^2z & x^2y \end{vmatrix} = i(x^2 - x^2) - k(\cancel{y}\ 2xy - 2xy) + k(2xz - 2xz)$

$= (0, 0, 0)$

$\therefore$ it is conservative.

$\therefore f_x = 2xyz$

$\int f_x = x^2yz + g(y, z)$

$\frac{d}{dy} = x^2z + g_y(y, z) = x^2z$

$\therefore g_y(y, z) = 0 \quad g(y, z) = \overset{C}{\cancel{0}} + h(z)$

$f = x^2yz + 0 + h(z)$

$\frac{d}{dz} f = x^2y + h'(z) \quad h'(z) = 0$

$\therefore f = x^2yz$

$r(t) = (t^2, \sin\frac{\pi t}{4}, e^{t^2-2t})$

~~$r(t) =$~~ $t = 0, \quad r(0) = (0, 0, 1)$

$t = 2, \quad r(2) = (4, \underset{1}{\cancel{\ }}, e^0) = (4, 1, 1)$

$f(\cancel{1, \frac{\sqrt{2}}{2}, e^{-1}}) - f(0, 0, 1) = 16$

$(4, 1, 1)$

Q19. let $F=\nabla f$, determin $\int_C F\,dr$.

$f=x^2y-z$ $\quad r_1=(t,t,0)\quad 0\le t\le 1$

$r_2=(t,t^2,0)\quad 0\le t\le 1.$

$\because\ f=x^2y-z\quad F=\nabla f=(2xy,x^2,-1)$

for r_1: when $t=0$, $r_1=P=(0,0,0)$

$t=1$, $r_1=p=(1,1,0)$

for r_2: when $t=0$, $r_2=(0,0,0)$

when $t=1$ $\quad r_2=(1,1,0)$

$\therefore\ \underset{(t=0)}{r_1=r_2}\quad \underset{(t=1)}{r_1=r_2}$

$\therefore\ F=\nabla f=(2xy,x^2,-1)$

$r_1=(t,t,0)\quad r'_1=(1,1,0)$

$r_2=(t,t^2,0)\quad r'_2=(1,2t,0)$

$\therefore\int_0^1(2xy,x^2,-1)\cdot(1,1,0)\,dt$

$=\int_0^1(2\cdot t\cdot t+t^2\cdot 1-1\cdot 0)\,dt$

$=\int_0^1 2t^2+t^2\,dt=\quad t^3\Big|_0^1=1$

$\int_0^1(2xy,x^2,-1)\cdot(1,2t,0)\,dt$

$=\int_0^1(2t^3,t^2,-1)\cdot(1,2t,0)\,dt$

$=\int_0^1 \cancel{2t^3+2t^5}\ 2t^3+2t^3\,dt$

$=\quad t^4\Big|_0^1=1$

$\therefore$ it is indeed 1.

for $f(Q)-f(p)$:

$\int_C F\,dr=f(Q)-f(p)$

$=\cancel{f(t,t^2,0)-f(t,t,0)}$ $\quad 1,2t,0 \quad 1,1,0$

$=f(1,1,0)-f(0,0,0)$ $\quad\therefore$ it is 1.

$=(1-0)-(0-0)=1$