

16.2

3. (a) $f(x, y, z) = (t^{-2}, t^2)$

$$dr = \langle 1, -t^{-2} \rangle dt$$

$$\int_C \langle t^{-2}, t^2 \rangle \cdot \langle 1, -t^{-2} \rangle$$

$$= t^{-2} + (t^2 \cdot (-t^{-2}))$$

$$= t^{-2} - t^0 = t^{-2} - 1$$

$$\int_1^2 (t^{-1} - 1) dt = -\frac{1}{2}$$

9. $f(x, y) = \sqrt{1+9xy}$ $y = x^2$ $0 \leq x \leq 1$

$$(x, x^2), (t, t^2) \quad 0 \leq t \leq 1$$

$$x' = 1 \quad y' = 2t \quad \sqrt{1+9t^2}$$

$$\int_0^1 \sqrt{1+9t^2} \cdot \sqrt{1+9t^2} dt$$

$$= 2.8$$

11. $f(x, y, z) = z^2$, $r(t) = (2t, 3t, 4t)$ for $0 \leq t \leq 2$

$$x = 2t, \quad y = 3t, \quad z = 4t \quad z^2 = 16t^2$$

$$x' = 2, \quad y' = 3, \quad z' = 4$$

$$\sqrt{4+9+16} = \sqrt{29}$$

$$\int_0^2 16t^2 \cdot \sqrt{29} dt = 229.8$$



13. $A(0,0,1)$ $B(0,2,0)$ $C(1,1,1)$

$$BC = (0,2,0) + t(1,1,1)$$

$$= (t, 2+t, t)$$

~~$$\sqrt{t^2 + (2+t)^2 + t^2}$$~~ $x=t$

$$x=t \quad y=2+t \quad z=t$$

$$x'=1 \quad y'=1 \quad z'=1$$

$$\sqrt{1+1+1} = \sqrt{3}$$

$$\int_0^1 t e^{t^2} \cdot \sqrt{3} dt = 1.488$$

17. $x=4t$ $y=-3t$ $z=12t$

$$x'=4 \quad y'=-3 \quad z'=12$$

$$\sqrt{16+9+144} = 13$$

$$\int_2^5 13 dt = \int_2^5 13 dt = 39$$

13 the distance between $(\Rightarrow 8, -6, 12)$ and

21. (x, x^2) on $x \in 2$ (t_1, t_2) at $t \in 2$ $(2a, 15, 60)$.

$$x=t \quad y=t^2 \quad x'=1 \quad y'=2t$$

$$dx=1dt \quad dy=2t dt$$

$$\int_0^2 t^2 dt - \int_0^2 2t^2 dt$$

$$= \int_0^2 t^2 - 2t^2 dt = -\frac{8}{3}$$



$$29. (0, 0, 0) + t(1, 4, 4) = (t, 4t, 4t).$$

$$\cancel{x'=1} \quad \cancel{y'=4}$$

$$dx = dt \quad dy = 4dt \quad dz = 4dt$$

$$\int_0^1 \cancel{4t dt} (t - 4t) dt + 4t(4t - 4t) dt + \cancel{4t dt}$$

$$= \frac{13}{2}$$

$$31. (1, 0) + t(-1, 1) = (1-t, t).$$

$$x = 1-t \quad y = t$$

$$dx = -dt \quad dy = dt$$

$$\int_0^1 \frac{1-t dt + (1-t) dt}{(1-t)^2 + t^2} dt = 1.5708$$

$$35. P(0, 0, 0) \quad Q = (-1, 1, 1).$$

$$R(0, 0, 0) + t(-1, 1, 1)$$

$$= (-t, t, t)$$

$$x = -t \quad y = t \quad z = t$$

$$x' = -1 \quad y' = 1 \quad z' = 1$$

$$\int_0^1 \sqrt{1+1+1} \sqrt{3} dt$$

$$= 6.701$$



11.3

1. $F = (y \sin yz) \mathbf{i} + \cancel{\sin yz} \mathbf{j} + xy^2 \cos yz \mathbf{k}$

$$\int_C F dr = f(1,1,\pi) - f(0,0,0) = 0$$

3. $(3, 6y) \Rightarrow \nabla = \nabla$

~~$f_x =$~~ $f(x,y) = 3x + 3y^2$

$f_x = 3 \Rightarrow f_y = 6y$

$F(x,y) = (3, 6y) \Rightarrow f(x,y) = 3x + 3y^2$ $r(t) = (t, 2t^{-1})$
 ~~$r(t) = (t, -2t)$~~

~~$\int_C F dr =$~~ $r(1) = (1, 2)$ $r(1/2) = (1/2, 1)$

$f(1/2, 1) - f(1, 2) = -\phi$

5. $f(x,y,z) = xy e^z$ $f_x = y e^z$

$f_y = x e^z$ $f_z = xy e^z$

$r(1) = (1, 1, 0)$ $r(2) = (4, 8, 1)$

$f(4, 8, 1) - f(1, 1, 0) = 85.985$

9. $F = y^2 \mathbf{i} + (xy + e^z) \mathbf{j} + ye^z \mathbf{k}$

$\frac{d}{dx}$ $\frac{d}{dy}$ $\frac{d}{dz}$

$f_x = y^2$
 $f = \int y^2 dx$

$2xy + f_y = xy e^z$

y^2 $xy e^z$ ye^z

$= xy^2 + g(y,z)$

$f_y = e^z$

$\nabla \cdot F = 0$ is conservative. $f_y = xy e^z$



$$g(y, z) = \cancel{h(z)} + e^{zy}$$

$$f = xy^2 + e^{zy} \cancel{h(z)}$$

$$f_z = ye^z$$

$$e^{zy} \neq 0 + h'(z) = ye^z$$

$$h'(z) = 0 \cdot h(z) = 0$$

$$f = xy^2 + e^{zy}$$

13. $f_x = z \sec^2 x$

$$f = \cancel{z \sec^2 x} \cdot z \tan x + g(y, z)$$

$$f_y = z$$

$$0 + g_y = z$$

$$g_y = z$$

$$g(y, z) = h(z) + \frac{z^2}{2} y$$

$$f_z = y + \tan x$$

$$f = z \tan x + zy + h(z)$$

$$y \cancel{h'(z)} = y \cancel{\tan x}$$

$$\tan x + y h'(z) = y + \tan x$$

$$\cancel{h'(z) = \tan x} \quad h'(z) = 0 \quad h'(z) = 0$$

$$h(z) = 0$$

$$f = z \tan x + zy$$



15. Is conservative.

$$f_x = 2xy + 5$$

$$f = 2y \frac{x^2}{2} + 5x + g(y, z)$$

$$= x^2 y + 5x + g(y, z)$$

$$f_y = x^2 - 4z$$

$$x^2 + g_y = x^2 - 4z$$

$$g_y = -4z \quad g(y, z) = h(z) - 4zy$$

$$f = x^2 y + 5x - 4yz + h(z)$$

$$f_z = -4y$$

$$-4y + h'(z) = 0$$

$$h'(z) = 0 \quad h(z) = c$$

$$f = x^2 y + 5x - 4yz$$

17. Is conservative

$$f_x = 2xyz \quad f = \frac{1}{2} x^2 yz + g(y, z)$$

$$f_y = x^2 z \quad \frac{1}{2} x^2 z + g_y = x^2 z$$

$$g_y = 0$$

$$g(y, z) = 0 \quad h(z) = 0 \quad (g(y, z) = h(z))$$

$$f = \frac{1}{2} x^2 yz \quad f = x^2 yz + h(z)$$

$$x^2 y + h'(z) = x^2 y$$

$$h'(z) = 0 \quad h(z) = 0$$

$$f = \frac{1}{2} x^2 yz$$



$$r(t) = (t^2, \sin(\frac{\pi t}{4}), e^{t^2 - 2t})$$

$$r(0) = (0, 0, 1)$$

$$r(2) = (4, 1, 1)$$

$$f(4, 1, 1) - f(0, 0, 1) = 16 - 0 = 16.$$

19. is conservative

$$r_1 = (t, t, 0)$$

$$r(0) = (0, 0, 0)$$

$$r(1) = (1, 1, 0)$$

$$f(1, 1, 0) - f(0, 0, 0) = 1 - 0 = 1.$$

$$r_2 = (t, t^2, 0)$$

$$r(0) = (0, 0, 0) \quad r(1) = (1, 1, 0)$$

$$f(1, 1, 0) - f(0, 0, 0) = 1.$$

