Math 251 Shaun Goda Section 23 HW # 10

16.2 :

3) a) $F(r(t)) = \langle t^{-2}, t^{2} \rangle$ $\quad dr = \langle 1, -t^{-2} \rangle dt$

b) $\langle t^{-2}, t^{2} \rangle \cdot \langle 1, -t^{-2} \rangle =$ [illegible] $= \boxed{t^{-2} - 1}$

$$\int_1^2 (t^{-2} - 1)\,dt = \left| -t^{-1} - t \right|_1^2 = \boxed{-\frac{1}{2}}$$

9) $$\int_C \sqrt{1 + 9xy}\,ds = \int_0^1 \sqrt{1+9t^4}\sqrt{1^2 + (3t^2)^2}\,dt$$
$$= \boxed{\frac{14}{5}}$$

11) $$\int_C z^2\,ds = \int_0^2 16t^2 \sqrt{2^2+3^2+4^2}\,dt = \sqrt{29}\left| \frac{16t^3}{3} \right|_0^2 = \boxed{\frac{128\sqrt{29}}{3}}$$

13) $$\int_C x e^{z^2}\,ds = \int_0^1 t e^{t^2}\,dt = \left| \frac{e^{t^2}}{2} \right|_0^1 = \boxed{\frac{e}{2}}$$ not so sure.

17) $$\int_C 1\,ds = \int_2^5 \sqrt{(4)^2 + (-3)^2 + (12)^2}\,dt = \left| 13t \right|_2^5 = \boxed{39}$$

this integral represent the distance between

(8, -6, 24) to (20, -15, 60)

27) $$\int_C y\,dx - x\,dy = \int_0^2 t^2\sqrt{1^2+(2t)^2}\,dt - \int_0^2 t\sqrt{1^2+(2t)^2}\,dt$$

$$\boxed{\approx 2.713}$$

29) $$\int_0^1 (t - 4t)\sqrt{t^2+16t^2+16t^2}\,dt + \int_0^1 0\,dy + \int_0^1 4t\sqrt{t^2+16t^2+16t^2}$$

$$= \boxed{\frac{\sqrt{33}}{3}}$$

31) $\int_C \frac{-y\,dx + x\,dy}{x^2+y^2} = \int_C \frac{-y}{x^2+y^2}\,dx + \int_C \frac{x}{x^2+y^2}\,dy$

$\vec{r}(t) = (1-t)\langle 1, 0\rangle + t\langle 0, 1\rangle = \langle 1-t, t\rangle \qquad 0 \le t \le 1$

$\int_0^1 \frac{t}{(1-t)^2+t^2}\,dt + \int_0^1 \frac{1-t}{(1-t)^2+t^2}\,dt = \boxed{\frac{\pi}{2}}$

35) $\int_0^1 \langle e^t, 1, t\rangle \cdot \langle 0, 0, 1\rangle\,dt$

$= \int_0^1 (\;)\,dt = \Big|2t\Big|_0^1 = 2$

$\int_0^1 \langle e^1, e^{-t}, e^t\rangle \cdot \langle 0, 1, 0\rangle\,dt$

$= \int_0^1 e^{-t}\,dt = \Big|-e^{-t}\Big|_0^1 = -e^{-1}$

$\int_0^1 \langle e, e^{-t-1}, e\rangle \cdot \langle -1, 0, 0\rangle\,dt$

$= \int_0^1 -e\,dt = \Big|-et\Big|_0^1 = -e$

$\boxed{2 - e^{-1} - e}$

16.3:

1) $\nabla F = \langle y\sin(yz), x\cos(zy)z, xy^2\cos(xz)\rangle$

$r(t) = (1-t)(0,0,0) + t(1,1,\pi) = (t, t, \pi t)$

$\nabla F \cdot r(t) = \langle t\sin(\pi t^2), \pi t^2\cos(\pi t^2), t^3\cos(\pi t^2)\rangle \cdot \langle 1, 1, \pi\rangle$

$= t\sin(\pi t^2) + \pi t^2\cos(\pi t^2) + \pi t^3\cos(\pi t^2)$

$\int_C \nabla F \cdot r(t)\,dr \approx 1.845$

3) $\nabla f(x,y) = \langle 3, 6y \rangle = F(x,y)$

$\int_C F\,dr = \int_1^4 \langle 3, 12t^{-1} \rangle \cdot \langle 1, -2t^{-2} \rangle\,dt$

$= \int_1^4 (3 - 24t^{-3})\,dt = -\frac{9}{4}$

5) $\nabla f(x,y,z) = \langle ye^z, xe^z, xye^z \rangle = F(x,y,z)$

$\int_C F\,dr = \int_1^2 \langle t^3 e^{t-1}, t^2 e^{t-1}, t^5 e^{t-1} \rangle \cdot \langle 2t, 3t^2, 1 \rangle\,dt$

$= \int_1^2 (2t^4 e^{t-1} + 3t^4 e^{t-1} + t^5 e^{t-1})\,dt = 32e - 1$

9) $P_y = 2y \quad Q_x = 2y \quad P_z = 0 \quad R_x = 0 \quad Q_z = e^z \quad R_y = e^z$

F is conservative since $P_y = Q_x$, $P_z = R_x$, $Q_z = R_y$

$\frac{\partial f}{\partial x} = y^2 \quad f(x,y,z) = xy^2 + g(y,z)$

$\frac{\partial f}{\partial y} = 2xy + \frac{\partial g}{\partial y} \quad \frac{\partial g}{\partial y} = e^z \quad g = ye^z$

$f(x,y,z) = xy^2 + ye^z$

13) $P_y = 0 \quad Q_x = 0 \quad P_z = \sec^2 x \quad R_x = \sec^2 x \quad Q_z = 1 \quad R_y = 1$

F is conservative since $P_y = Q_x$, $P_z = R_x$, $Q_z = R_y$

$\frac{\partial f}{\partial x} = z\sec^2 x \quad f(x,y,z) = z\tan x + g(y,z)$

$\frac{\partial f}{\partial y} = 0 + \frac{\partial g}{\partial y} \quad \frac{\partial g}{\partial y} = z \quad g = zy$

$f(x,y,z) = z\tan x + zy$

15) $P_y = 2x$ $\quad Q_x = 2x$ $\quad P_z = 0$ $\quad R_x = 0$ $\quad Q_z = -4$ $\quad R_y = -4$

F is conservative since $P_y = Q_x$, $P_z = R_x$, $Q_z = R_y$

$\frac{\partial f}{\partial x} = 2xy + 5$ $\quad f(x,y,z) = 2x^2y + 5x + g(y,z)$

$\frac{\partial f}{\partial y} = 2x^2 + g_y(y,z)$ $\quad g_y(y,z) = -x^2 - 4z$ $\quad g(y,z) = -x^2y - 4zy$

$f(x,y,z) = x^2y + 5x - 4zy$

17) $\int_C 2xyz\,dx + x^2z\,dy + x^2y\,dz$

$\int_0^2 2(t^2)(\sin(\pi\frac{t}{4}))(e^{t^2-2t})(2t)\,dt = 0.22779$

$\int_0^2 (t^2)^2(e^{t^2-2t})(\frac{\pi}{4}\cos(\frac{\pi}{4}x))\,dt = 0.641923$

$\int_0^2 (t^2)^2(\sin(\frac{\pi}{4}t))(2x-2)e^{x^2-2x}\,dt = 6.0538$

$0.22779 + 0.641923 + 6.0538 = 6.92$

19) $\vec{r}(t) = (1-t)\langle t, t, 0\rangle + t\langle t, t^2, 0\rangle = \langle t, t-t^2+t^3, 0\rangle$

$\int_C (x^2y - z)\,dr$

$= \int_0^1 t^2(t-t^2+t^3)\sqrt{1^2+(1-2t+3t^2)^2}\,dt \approx 0.3821$