HW due 11/15/20

16.2 3, 9, 11, 13, 17, 27, 29, 31, 35

16.3 1, 3, 5, 9, 13, 15, 17, 19

16.2

3a. $F = \langle y^2, x^2 \rangle \quad y = x^{-1} \quad 1 \le x \le 2$

$r(t) = (t, t^{-1}) \quad F(r(t)) = \langle t^{-2}, t^2 \rangle$

$dr = \langle 1, -\frac{1}{t^2} \rangle dt$

b. $\langle t^{-2}, t^2 \rangle \cdot \langle 1, -\frac{1}{t^2} \rangle dt$

$= (t^{-2} - 1)\, dt = \left(\frac{1}{t^2} - 1\right) dt$

$\int_1^2 \left(\frac{1}{t^2} - 1\right) dt = -\frac{1}{t} - t \Big|_1^2 = -\frac{5}{2} - (-1-1) = \boxed{-\frac{1}{2}}$

9. $f(x,y) = \sqrt{1+9xy} \quad y = x^3 \quad 0 \le x \le 1$

$x = t \quad y = t^3$

$f(r(t)) = \sqrt{1+9t^4} \qquad r'(t) = \langle 1, 3t^2 \rangle$

$\|r'(t)\| = \sqrt{1+9t^4}$

$\int_0^1 \sqrt{1+9t^4}\sqrt{1+9t^4}\, dt = \int_0^1 1 + 9t^4\, dt = t + \frac{9}{5}t^5 \Big|_0^1$

$= \frac{14}{5} = \boxed{2.8}$

11. $f(r(t)) = 16t^2 \quad r'(t) = \langle 2, 3, 4 \rangle$

$\|r'(t)\| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$

$\int_0^2 16t^2(\sqrt{29})\, dt = 16\sqrt{29}\left(\frac{t^3}{3}\right)\Big|_0^2 = \boxed{\frac{128\sqrt{29}}{3}}$

13. $r(t) = (1-t)\langle 0, 0, 1 \rangle + t\langle 0, 2, 0 \rangle$

$= \langle 0, 0, 1-t \rangle + \langle 0, 2t, 0 \rangle = \langle 0, 2t, 1-t \rangle \quad 0 \le t \le 1$

C. $c(t) = \langle 0, 2t, 1-t \rangle \quad c'(t) = \langle 0, 2, -1 \rangle$

$\|c'(t)\| = \sqrt{0^2 + 2^2 + (-1)^2} = \sqrt{5}$

$f(c(t)) = 0\, e^{(\dots)^2} = 0$

$\int_0^1 0 \cdot \sqrt{5}\, dt = \boxed{0}$

17. $r(t) = \langle 4t, -3t, 12t \rangle \quad 2 \le t \le 5$

$r'(t) = \langle 4, -3, 12 \rangle \quad \|r'(t)\| = \sqrt{4^2 + (-3)^2 + 12^2} = 13$

$\int_2^5 13\, dt = 13t \Big|_2^5 = 65 - 26 = \boxed{39}$

27. $c(t) = (t, t^2) \quad 0 \le t \le 2$

$c'(t) = \langle 1, 2t \rangle$

$\langle t^2, -t \rangle \cdot \langle 1, 2t \rangle = t^2 - 2t^2 = -t^2$

$-\int_0^2 t^2\, dt = -\frac{t^3}{3}\Big|_0^2 = \boxed{-\frac{8}{3}}$

29. $c(t) = (t, 4t, 4t) \quad 0 \le t \le 1$

$c'(t) = \langle 1, 4, 4 \rangle$

$\int_0^1 (-3t, 0, 4t) \cdot (1, 4, 4)\, dt = \int_0^1 -3t + 16t\, dt$

$= \int_0^1 13t\, dt = \frac{13}{2}t^2 \Big|_0^1 = \boxed{\frac{13}{2}}$

31. $c(t) = (1-t, t) \quad 0 \le t \le 1$

$dx = -dt \quad dy = dt$

$\int_0^1 \frac{-t(-dt) + (1-t)dt}{(1-t)^2 + t^2} = \int_0^1 \frac{dt}{2t^2 - 2t + 1} = \frac{1}{2}\int_0^1 \frac{dt}{t^2 - t + \frac{1}{2}}$

$= \frac{1}{2}\int_0^1 \frac{dt}{(t-\frac{1}{2})^2 + (\frac{1}{2})^2} \qquad u = t - \frac{1}{2} \quad du = 1dt$

$= \frac{1}{2}\left[\frac{1}{1/2}\tan^{-1}\left(\frac{u}{1/2}\right)\right]_{-1/2}^{1/2}$

$= \tan^{-1}(1) - \tan^{-1}(-1) = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \boxed{\frac{\pi}{2}}$

35. $\int_{PA} F\, ds = \int_0^1 1\, dt = t\Big|_0^1 = 1$

$\int_{AB} F\, ds = \int_0^1 e^{-t}\, dt = -e^{-t}\Big|_0^1 = -e^{-1} + 1$

$\int_{BQ} F\, ds = \int_0^1 -e\, dt = -e(t)\Big|_0^1 = -e$

$\int_C F\, ds = \boxed{2 - e^{-1} - e}$

16.3

1. $f(x,y,z) = xy\sin z$

$f(1,1,\pi) = (1)(1)\sin(\pi) = 0$

$f(0,0,0) = (0)(0)\sin(0) = 0$

$\int_C F\, dr = \boxed{0}$

3. $f(x,y) = 3x + 3y^2 \quad \nabla f = \langle 3, 6y \rangle = F(x,y)$

$F(r(t)) = \langle 3, 12t^{-1} \rangle \quad r'(t) = \langle 1, -\frac{2}{t^2} \rangle$

$\int_C F\, dr = \int_1^4 \langle 3, 12t^{-1} \rangle \cdot \langle 1, -\frac{2}{t^2} \rangle dt = \int_1^4 3 - \frac{24}{t^3}\, dt$

$= 3t + \frac{12}{t^2}\Big|_1^4 = \left(12 + \frac{12}{16}\right) - (3 + 12) = \boxed{-\frac{9}{4}}$

16.3: 5, 9, 13, 15, 17, 19

5. $f(x,y,z) = xye^z \quad \nabla f = \langle ye^z, xe^z, xye^z \rangle = F$

$r(t) = (t^2, t^3, t-1) \quad 1 \le t \le 2$

$F(r(t)) = \langle t^3 e^{t-1}, t^2 e^{t-1}, t^5 e^{t-1} \rangle \quad r'(t) = \langle 2t, 3t^2, 1 \rangle$

$F(r(t)) \cdot r'(t) = 2t^4 e^{t-1} + 3t^4 e^{t-1} + t^5 e^{t-1} = 5t^4 e^{t-1} + t^5 e^{t-1}$

$\int_C F\,dr = \int_1^2 5t^4 e^{t-1} + t^5 e^{t-1}\,dt = t^5 e^{t-1} \Big|_1^2 = \boxed{32e - 1}$

9. $F = y^2 i + (2xy + e^{z}) j + ye^z k$

$V(x,y,z) = xy^2 + f(y,z)$

$\frac{dV}{dy} = 2xy + \frac{df}{dy} = 2xy + e^z \qquad \frac{df}{dy} = e^z$

$f(y,z) = ye^z + g(z) \qquad \frac{dV}{dz} = ye^z + \frac{dg}{dz} = ye^z$

$\frac{dg}{dz} = 0 \quad g(z) = 0$

$\boxed{V(x,y,z) = xy^2 + ye^z}$

13. $F = \langle z\sec^2 x, z, y + \tan x \rangle$

$V(x,y,z) = z\tan x + f(y,z)$

$\frac{dV}{dy} = \frac{df}{dy} = z \qquad f(y,z) = yz + g(z)$

$V(x,y,z) = z\tan x + yz + g(z)$

$\frac{dV}{dz} = \tan x + y + g'(z) = \tan x + y \qquad g'(z) = 0$

$g(z) = 0 \quad \boxed{V(x,y,z) = z\tan x + yz}$

15. $F = \langle 2xy + 5, x^2 - 4z, -4y \rangle$

$V(x,y,z) = x^2 y + 5x + f(y,z)$

$\frac{dV}{dy} = x^2 + \frac{df}{dy} = x^2 - 4z \qquad \frac{df}{dy} = -4z$

$f(y,z) = -4zy + g(z)$

$V(x,y,z) = x^2 y + 5x - 4zy + g(z)$

$\frac{dV}{dz} = -4y + g'(z) \qquad g'(z) = 0 \qquad g(z) = 0$

$\boxed{V(x,y,z) = x^2 y + 5x - 4zy}$

17. $\int_C 2xyz\,dx + x^2 z\,dy + x^2 y\,dz$

$r(t) = (t^2, \sin(\frac{\pi t}{4}), e^{t^2 - 2t}) \qquad 0 \le t \le 2$

$r'(t) = \langle 2t, \frac{\pi}{4}(\cos(\frac{\pi t}{4})), (2t-2)e^{t^2-2t} \rangle$

$\int_0^2 (2t^2 \sin(\frac{\pi t}{4}) e^{t^2-2t})(2t), (t^4 e^{t^2-2t})(\frac{\pi}{4}\cos\frac{\pi t}{4}),$

$(t^4 \sin(\frac{\pi t}{4}))((2t-2)e^{t^2-2t})\,dt$

$= t^4 \sin(\frac{\pi t}{4}) e^{t^2-2t} \Big|_0^2 = 16\sin\frac{\pi}{2} = 16(1) = \boxed{16}$

19. $f = x^2 y - z \quad r_1(t) = \langle t, t, 0 \rangle \quad r_2(t) = \langle t, t^2, 0 \rangle$

$\int_{C_1} F\,dr = f(r_1(1)) - f(r_1(0))$

$= f(1,1,0) - f(0,0,0) = 1 - 0 = 1$

$\int_{C_2} F\,dr = f(r_2(1)) - f(r_2(0))$

$= f(1,1,0) - f(0,0,0) = 1 - 0 = 1$

$\boxed{1 = 1 \text{ both equal}}$