16.2 & 16.3 (Nov. 15th)

16.2: # 3, 9, 11, 13, 17, 27, 29, 31, 35

16.3: # 1, 3, 5, 9, 13, 15, 17, 19

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3) a) $\vec{F} = \langle y^2, x^2 y \rangle$

$\vec{r}(t) = \langle t, t^{-1} \rangle$

$\vec{F}\langle x, y \rangle = \langle \frac{1}{t^2}, t^2 \rangle$

$\boxed{\vec{F}(r(t)) = \langle \frac{1}{t^2}, t^2 \rangle}$

$\boxed{dr = \langle 1, -t^{-2} \rangle dt}$

b) $\langle \frac{1}{t^2}, t^2 \rangle \cdot \langle 1, -t^{-2} \rangle$

$F(r(t)) \cdot dr = \langle \frac{1}{t^2}, -1 \rangle$

$\int_1^2 (t^{-1} - 1)\, dt$

$= \boxed{-\frac{1}{2}}$

9) $x = t$, $y = t^3$; $0 \le t \le 1$

$c(t) = \langle t, t^3 \rangle$, $0 \le t \le 1$

$f(x, y) = \sqrt{1 + 9xy}$

$\int_0^1 \sqrt{1 + 9t^4} \, \| \langle 1, 3t^2 \rangle \| dt$

$= \int_0^1 \sqrt{1 + 9t^4} \cdot \sqrt{1 + 9t^4}\, dt$

$= (t + \frac{9}{5} t^5) \Big|_0^1$

$= 14/5 = 2.8$

11) $c'(t) = \frac{d}{dt}(2t, 3t, 4t)$

$\| c'(t) \| = \sqrt{29}$

$z = 4t$, $f(c(t)) = 16t^2$

$\int_0^2 f(c(t)) \, \| c'(t) \| dt$

$= \int_0^2 16t^2 \sqrt{29}\, dt$

$= \sqrt{29}\, 16 \frac{t^3}{3} \Big|_0^2$

$= \boxed{\frac{128\sqrt{29}}{3}}$

13) $c'(t) = (0, 2, -1)$

$f(x,y,z) = xe^{z^2}$

$f(c(t)) = f(0, 2t, 1-t) = 0$

$\int_0^1 0 \|c'(t)\| dt = 0$

$\overrightarrow{BC} = c(t) = (t, 2-t, t)$

$c'(t) = (1, -1, 1)$

$\|c'(t)\| = \sqrt{3}$

$f(c(t)) = f(t, 2-t, t) = te^{t^2}$

$\int_0^1 te^{t^2}\sqrt{3}\, dt$

$\sqrt{3}\int_0^1 e^u \frac{du}{2}$

$= \frac{\sqrt{3}}{2}[e^u]_0^1$

$= \frac{\sqrt{3}}{2}(e-1)$

$\overrightarrow{CA} = c(t) = (A-C)t + C$

$c'(t) = (1, 1, 1)$

$\|c(t)\| = \sqrt{3}$

$\int_0^1 (1-t)(e)\, dt = e\int_0^1 (1-t)$

$e[t - \frac{t^2}{2}]_0^1 = \frac{e}{2}$

$\int_c f(x,y,z)\, ds = 0 + \frac{\sqrt{3}}{2}(e-1) = \boxed{\frac{\sqrt{3}}{2}(e-1)}$

17) $c'(t) = \frac{d}{dt}\langle 4t, -3t, 12t\rangle$

$= (4, -3, 12)$

$\|c'(t)\| = \sqrt{4^2 + (-3)^2 + 12^2} = 13$

$\int_2^5 \|c'(t)\| dt$

$= \int_2^5 13\, dt$

$= 13(5-2)$

$= \boxed{39}$

27) $dy = 2x\, dx$

$y\,dx - x\,dy = x^2 dx - x(2x\,dx)$

$= x^2 dx - 2x^2 dx$

$= (x^2 - 2x^2) dx$

$= -x^2 dx$

$\int_0^2 -x^2 dx$

$= -\int_0^2 x^2 dx$

$-\left[\frac{x^3}{3}\right]_0^2$

$-\left(\frac{2^3}{3}\right)$

$\boxed{-8/3}$

29) $x = t, y = 4t, z = 4t, \quad 0 \le t \le 1$

$dx = dt, dy = 4dt, dz = 4dt$

$\int_0^1 (t + 4t)dt + (4t - 4t)dt + 4t\, 4dt$

$\int_0^1 13t\, dt$

$13(\frac{1}{2})t^2 ]_0^1$

$= \boxed{\frac{13}{2}}$

31) $(x, y) = (t, 1-t)$

$\int \frac{-(1-t)dt + t\,d(1-t)}{t^2 + (1-t)^2}$

$= \int_0^1 \frac{-1}{2t^2 - 2t + 1} dt$

$= \int_0^1 \frac{1}{2((t - \frac{1}{2})^2 + \frac{1}{4})} dt$

$= \frac{1}{2} \cdot 2 \left(\arctan\left(\frac{t - \frac{1}{2}}{1/2}\right)\right)\Big|_0^1$

$= \boxed{\pi/2}$

35) $F \cdot dr = e^0\,dz = dz$

$F \cdot dr = e^{-y}\,dy = e^{-y}\,dy$

$F \cdot dr = e^1\,dx = e\,dx$

$\int_0^1 dz + \int_0^1 e^{-y}\,dy + \int_0^1 e^1\,dx$

$= z\Big|_0^1 + \dfrac{e^{-y}}{-1}\Big|_0^1 + ex\Big|_0^1$

$= 1\,e^{-1} - e + 1$

$= \boxed{2 - e - \frac{1}{e}}$

16.3: ~~#~~ 1, 3, 5, 9, 13, 15, 17, 19

1) $\int_C \vec{F} \cdot d\vec{r} = \int_C \nabla f \cdot dr$

$= f(r_2) - f(r_1) = f(1,1,\pi) - f(0,0,0)$

$= (1)(1)\sin(\pi) - (0)(0)\sin(0)$

$= \boxed{0}$

3) $r(t) = \langle t, \frac{2}{t} \rangle,\ r'(t) = \langle 1, -\frac{2}{t^2} \rangle$

$F(r(t)) = \langle 3, \frac{12}{t} \rangle$

$\int_1^4 \langle 3, \frac{12}{t} \rangle \cdot \langle t, -\frac{2}{t^2} \rangle dt$

$= \int_1^4 3 - \dfrac{24}{t^3}\,dt$

$= 12 + \frac{12}{16} - (3+12) = \boxed{-\frac{9}{4}}$

5) $\nabla v = \dfrac{\partial (yxe^z)}{\partial x} + \left(\dfrac{\partial yxe^z}{\partial y} + \dfrac{\partial (yxe^z)}{\partial z}\right)$

$= ye^z i + xe^z j + yxe^z k$

$c(t) = \langle t^2, t^3, t-1 \rangle$

$dr = \langle 2t, 3t^2, 1 \rangle$

$F \cdot dr = \int_1^2 (t^3e^{t-1}, t^2e^{t-1}, t^5e^{t-1})(2t, 3t^2, 1)\,dt$

$= [t^5 e^{t-1}]\Big|_1^2$

$= \boxed{32e - 1}$

9) $\vec{F} = y^2 i + (2xy + e^2 j) + ye^2 k$

$\text{curl} F = i(e^2 - e^2) - j(0-0) + k(4-4)$

$= 0$

The vector field is conservative on $\mathbb{R}^3$

$\nabla \phi = F$

$\left(i \frac{\partial \phi}{\partial x} + j \frac{\partial \phi}{\partial y} + k \frac{\partial \phi}{\partial z}\right) = y^2 \hat{i} + (2xy + e^2)\hat{j} + ye^2 k$

$\phi = xy^2 + C_1$

$\phi = xy^2 + ye^2 + C_2$

$\phi = ye^2 + C_3$

$\boxed{\phi(x,y,z) = xy^2 + ye^2 + C}$

13) $\vec{F} = \langle z\sec^2 x, z, y + \tan x \rangle$

$\text{curl} F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z\sec^2 x & z & y + \tan x \end{vmatrix}$

$= i(1-1) + \hat{j}(\sec^2 x - \sec^2 x) + \hat{k}(0-0)$

$= 0 \rightarrow \vec{F}$ is conservative

$\nabla f = \vec{F}$

$\left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle = \langle z\sec^2 x, z, y + \tan x \rangle$

$\frac{\partial f}{\partial x} = z\sec^2 x, \quad \frac{\partial f}{\partial y} = z, \quad \frac{\partial f}{\partial z} = y + \tan x$

$f = z\tan x + g_1(y,z), \quad f = yz + g_2(x,z)$

$\boxed{f = z(y + \tan x)}$

15) $\vec{F} = (2xy+5)\hat{i} + (x^2-4z)\hat{j} - 4y\hat{k}$

$\text{curl } \vec{F} = \hat{i}(-4+4) - \hat{j}(0-0) + \hat{k}(2x-2x)$

$= 0 \rightarrow F$ is conservative

$(2xy+5)\hat{i} + (x^2-4z)\hat{j} - 4y\hat{k} = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$

$\frac{\partial \phi}{\partial x} = 2xy+5$ $\qquad \frac{\partial \phi}{\partial y} = x^2-4z$ $\qquad \frac{\partial \phi}{\partial z} = -4y$

$\int 2xy+5$ $\qquad \int (x^2-4z)\,dy$ $\qquad = \int -4y\,dz$

$= x^2y+5x$ $\qquad = x^2y-4yz$ $\qquad = -4yz$

$\boxed{= x^2y+5x-4yz}$

17) $c(t) = \langle t^2, \sin(\frac{\pi t}{4}), e^{t^2-2t} \rangle, 0 \le t \le 2$

$\frac{\partial}{\partial x} f = 2xyz$

$= \int 2xyz\,dx$

$= x^2yz + g(y,z)$

$\frac{\partial f}{\partial y} f = x^2z + g'(y,z)$

$g'(y,z) = 0$

integrate w.r.t $y$, $g(y,z) = h(z)$

$f = x^2yz + h(z)$

$\frac{\partial f}{\partial z} = x^2y + h'(z)$, $x^2y = x^2y + h'(z)$, $h'(z) = 0$

$f = x^2yz + k$

$c(t) = \langle 0^2, \sin(0), e^0 \rangle = \langle 0, 0, 1 \rangle$

$c(2) = \langle 2^2, \sin\frac{\pi(2)}{4}, e^{4-4} \rangle = \langle 4, 1, 1 \rangle$

$\int = f\langle 4,1,1 \rangle - f\langle 0,0,1 \rangle$

$= \boxed{16}$

19) $r_1 = \langle t, t, 0 \rangle,\ 0 \le t \le 1$

$t=0,\ r_0 = \langle 0,0,0 \rangle$

$t=1,\ r(1) = \langle 1,1,0 \rangle$

$f(1,1,0) - f(0,0,0)$

$= 1 - 0 = \boxed{1.}$

$f(q) = 1$

$f(p) = 0.$