SHU BIN XIE
section22
11. 10. 2020.

3. Let $F=\langle y^2, x^2\rangle$, and Let C be the curve $y=x^{-1}$ for $1\le x\le 2$, oriented from left to right.

(a) calculate $F(r(t))$ and $dr=r'(t)dt$ for the parametrization of C given by $r(t)=(t, t^{-1})$.

?(b) calculate the dot product $F(r(t)) \cdot r'(t)dt$ and evaluate $\int_C F\cdot dr$.

(a) ~~$y=t$~~

$x=t \quad y=t^{-1}$

$F(r(t))=\langle t^{-2}, t^{2}\rangle$

$dr=r'(t)dt=\langle 1, -t^{-2}\rangle dt$

?(b) $F(r(t)) \cdot r'(t)dt$

$=\langle t^{-2}, t^{2}\rangle \cdot \langle 1, -t^{-2}\rangle$

$= t^{-2} - t^{0}$

$= t^{-2}-1$

$\int_1^2 (t^{-1}-1)dt = -\frac{1}{2}$.

$\int_C F\,dr$

$r(t)=(t, t^{-1})$

$x=t, \; y=t^{-1}$

$F=\langle t^{-2}, t^{2}\rangle=\sqrt{t^4+t^4}$

$y=x^{-1}. \quad 1\le x\le 2$

$1\le t\le 2$

$dr=\sqrt{1+t^4}$

$\int_1^2 \sqrt{t^4+t^4}\cdot\sqrt{1+t^4}\,dt = 2.675$.

9. compute $\int_C f\,ds$ for the curve specified.

$f(x,y) = \sqrt{1+9xy}$, $y = x^3$ for $0 \le x \le 1$.

~~dx=~~ $x = t$ ~~dt~~ $y = t^3$ $0 \le t \le 1$.

$dx = dt$ $dy = 3t^2\,dt$.

$ds = \sqrt{1^2 + (3t^2)^2}\,dt = \sqrt{1+9t^4}\,dt$.

$\int_0^1 \sqrt{1+9t^4}\ \sqrt{1+9t^4}\,dt$

$= \int_0^1 1+9t^4\,dt$

$= t + \frac{9}{5}t^5 \Big|_0^1$

$= 1 + \frac{9}{5}$

$= \frac{14}{5}$.

Ans: $\frac{14}{5}$.

11. $f(x,y,z) = z^2$, $r(t) = (2t, 3t, 4t)$ for $0 \le t \le 2$.

$x = 2t$ $y = 3t$ ~~dz~~ $z = 4t$

$dx = 2dt$ $dy = 3dt$ $dz = 4dt$

$0 \le t \le 2$.

$ds = \sqrt{2^2+3^2+4^2}\,dt\ \sqrt{4+9+16}\,dt = \sqrt{29}\,dt$.

$\int_0^2 (4t)^2 \sqrt{29}\,dt$

$= \sqrt{29} \int_0^2 16t^2\,dt$

$= \sqrt{29} \cdot \frac{16}{3}t^3 \Big|_0^2 = \sqrt{29} \times \frac{16}{3} \times 8 = \frac{128}{3}\sqrt{29}$.

Ans: $\frac{128}{3}\sqrt{29}$.

13. $f(x,y,z)=xe^{z^2}$, piecewise linear path from $(0,0,1)$ to $(0,2,0)$ to $(1,1,1)$.

AB: $(0,0,1)+t(0,2,-1)=(0,2t,1-t)$ $\quad 0\le t\le 1$

$x=0 \quad y=2t \quad z=1-t$ calculate mistake. ~~lose one~~

$dx=0 \quad dy=2dt \quad dz=-dt$.

$ds=\sqrt{0^2+4+1}\,dt=\sqrt{5}\,dt$

$\int_0^1 0\cdot e^{(1-t)^2}\cdot\sqrt{5}\,dt$

$=\int_0^1 dt$

$=t|_0^1$

$=\cancel{1}=0$

~~final Ans: $0+\frac{\sqrt{3}}{2}(e-1)$~~

~~$=\frac{\sqrt{3}}{2}(e-1)$~~

BC:

$(0,2,0)+t(1,-1,1)$

$=(t,2-t,t)$

$x=t \quad y=2-t \quad z=t$

$dx=dt \quad dy=-dt \quad dz=dt$

$ds=\sqrt{1+1+1}\,dt=\sqrt{3}dt$.

$\int_0^1 t\cdot e^{t^2}\sqrt{3}\,dt$

$=\frac{\sqrt{3}}{2}(e-1)$

BC: $(0,2,0)+t(1,-1,1)=(t,2-t,t)$ $\quad 0\le t\le 1$.

$x=t \quad y=2-t \quad z=t$

$dx=dt \quad dy=-dt \quad dz=dt$.

final Ans: $\frac{\sqrt{3}}{2}(e-1)+0$

$=\frac{\sqrt{3}}{2}(e-1)$

Ans: $\frac{\sqrt{3}}{2}(e-1)$

$ds=\sqrt{1^2+(-1)^2+1^2}\,dt=\sqrt{3}\,dt$.

$\int_0^1 te^{t^2}\sqrt{3}\,dt \qquad \int \frac{1}{2}\sqrt{3}\,e^u\,du$

~~$=e^{t^2+1}\cdot\frac{t}{t^2+1}\cdot\sqrt{3}\Big|_0^1$~~

$u=t^2$

$du=2t\,dt$

$\frac{1}{2}du=t\,dt$

$=\frac{\sqrt{3}}{2}e^u$

$=\frac{\sqrt{3}}{2}e^{t^2}\Big|_0^1$

$=\frac{\sqrt{3}}{2}(e-1)$

final Ans!

$\frac{\sqrt{3}}{2}e-\frac{\sqrt{3}}{2}+1$

~~$=\frac{\sqrt{3}}{2}e+\frac{2-2\sqrt{3}}{2}$~~

~~$\boxed{\text{Ans: } \frac{\sqrt{3}}{2}e+\frac{2-\sqrt{3}}{2}}$~~

17、Calculate $\int_C 1\,ds$, where the curve C is parametrized by $r(t)=(4t, -3t, 12t)$ for $2\le t\le 5$. What does this integral represent?

$x=4t \quad y=-3t \quad z=12t$

$dx=4dt \quad y=-3dt \quad z=12dt$.

$ds=\sqrt{16+9+144}\,dt=13dt$.

$\int_2^5 13dt = 13t|_2^5 = 13\times 3 = 39$.

Ans: 39. the distance between $(8,-6,24)$ and $(20,-15,60)$.

27、$\int_C y\,dx - x\,dy$, parabola $y=x^2$ for $0\le x\le 2$.

$x=t \quad y=t^2. \qquad 0\le t\le 2$.

$dx=dt \quad dy=2tdt$.

$\int_0^2 t^2 \cdot dt - t\cdot 2tdt$

$=\int_0^2 t^2dt - 2t^2dt$

$=\int_0^2 -t^2dt$

$=-\frac{1}{3}t^3|_0^2$

$=-\frac{1}{3}\times 8$

$=-\frac{8}{3}$. Ans: $-\frac{8}{3}$.

29. $\int_C (x-y)dx+(y-z)dy+z\,dz$, lane segment from $(0,0,0)$ to $(1,4,4)$.

lane: $(0,0,0)+t(1,4,4)=(t,4t,4t)$ $0\le t\le 1$.

$x=t$ $y=4t$ $z=4t$

$dx=dt$ $dy=4dt$ $dz=4dt$.

$\int_0^1 (t-4t)dt+(4t-4t)4dt+4t\cdot 4dt$.

$=\int_0^1 -3t\,dt+0+16t\,dt$

$=\int_0^1 13t\,dt$

$=\frac{13}{2}t^2\Big|_0^1$

$=\frac{13}{2}$ Ans: $\frac{13}{2}$.

31. $\int_C \frac{-y\,dx+x\,dy}{x^2+y^2}$, segment from $(1,0)$ to $(0,1)$

lane: $(1,0)+t(-1,1)=(1-t,t)$ $0\le t\le 1$.

$x=1-t$ $y=t$

$dx=-dt$ $dy=dt$.

$\int_0^1 \frac{t\,dt+(1-t)dt}{(1-t)^2+t^2}=\frac{dt}{1-2t+t^2+t^2}=\frac{dt}{2t^2-2t+1}=\frac{\pi}{2}$.

Ans: $\frac{\pi}{2}$

(I use maple calculate).

35, In exercises 35 calculate the line integral of $F(x,y,z) = \langle e^z, e^{x-y}, e^y \rangle$ over the given path. The blue path from P to Q in Figure 14.

P(0,0,0) (0,0,1) (0,1,1) Q(−1,1,1)

① line: $(0,0,0) + t(0,0,1) = (0,0,t) \quad 0 \le t \le 1$.

$x=0 \quad y=0 \quad z=t$

$dx=0 \quad dy=0 \quad dz=dt$.

$F(x,y,z) = \langle e^t, e^0, e^0 \rangle = \langle e^t, 1, 1 \rangle$

$\int_0^1 e^t \cdot 0 + 1 \cdot 0 + 1 \cdot dt$

$= 1$.

② line: $(0,0,1) + t(0,1,0) = (0,t,\cancel{1}0)$

$x=0 \quad y=t \quad z=0$

$dx=0 \quad dy=dt \quad dz=0$

$F(x,y,z) = \langle e^0, e^{-t}, e^t \rangle = \langle 1, e^{-t}, e^t \rangle$

$\int_0^1 1 \cdot 0 + e^{-t} \cdot dt + e^t \cdot 0$

$= -e^{-t} \Big|_0^1$

$= -e^{-1} + e^0$

$= 1 - e^{-1}$.

③ line $(0,1,1) + t(-1,0,0) = (-t,1,1)$.

$x=-t \quad y=1 \quad z=1$

$dx=-dt \quad dy=0 \quad dz=0$.

$F(x,y,z) = \langle e, e^{-t-1}, e^1 \rangle$

$\int_0^1 e\,(-dt) + e^{-t-1} \cdot 0 + e^1 \cdot 0$

$= -et \Big|_0^1$

$= -e$.

Final: $1 + 1 - e^{-1} - e$

$= 2 - e - \frac{1}{e}$.

Ans: $2 - e - \frac{1}{e}$.

homework 16.3

SHUBIN XIE
section 22
11.11.2020

1. Let $f(x,y,z) = xy\sin(yz)$ and $F=\nabla f$. Evaluate $\int_C F\cdot dr$ where C is any path from $(0,0,0)$ to $(1,1,\pi)$

$$F=\nabla f = \langle y\sin(yz),\ x\sin(yz)+xyz\cos(yz),\ xy^2\cos(yz)\rangle$$

$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ y\sin(yz) & x\sin(yz)+xyz\cos(yz) & xy^2\cos(yz) \end{vmatrix}$$

$i\left(\frac{d}{dy}(xy^2\cos(yz)) - \frac{d}{dz}(x\sin(yz)+xyz\cos(yz))\right) - j\left(\frac{d}{dx}(xy^2\cos(yz)) - \frac{d}{dz}(y\sin(yz))\right) + k\left(\frac{d}{dx}(x\sin(yz)+xyz\cos(yz)) - \frac{d}{dy}(y\sin(yz))\right)$

$= i*0 - 0*j + 0*k = 0$

F is conservative.
Therefore, we can use the fundamental theorem of line integrals to calculate this integrals.

$f(1,1,\pi) - f(0,0,0) = \sin\pi - 0 = 0-0=0.$

$\boxed{\text{Ans: } 0}$

3. $F(x,y) = \langle 3, 6y\rangle$, $f(x,y) = 3x+3y^2$, $r(t) = \langle t, 2t^{-1}\rangle$ for $1\le t\le 4$.

$f_x = 3 \quad f_y = 6y$

$F(x,y) = \langle 3, 6y\rangle \rightarrow$ correct.

$\int_C F(x,y) = f(x,y)\Big|_C$

$C: \ 1\le t\le 4$

$r(1) = \langle 1, 2\rangle$

$r(4) = \langle 4, \frac{1}{2}\rangle$.

$f(4,\frac{1}{2}) - f(1,2)$
$= 3\times4 + 3\times(\frac{1}{2})^2 - (3\times1 + 3\times2^2)$
$= 12 + \frac{3}{4} - (3+12)$
$= 12 - 15 + \frac{3}{4} = -3 + \frac{3}{4} = -\frac{9}{4}$

Ans: $-\frac{9}{4}$.

5. $F(x,y,z) = ye^z i + xe^z j + xye^z k$ · $f(x,y,z) = xye^z$

$r(t) = (t^2, t^3, t-1)$ for $1 \le t \le 2$.

$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ ye^z & xe^z & xye^z \end{vmatrix}$$

$i\left(\frac{d}{dy}(xye^z) - \frac{d}{dz}(xe^z)\right) - j\left(\frac{d}{dx}(xye^z) - \frac{d}{dz}(ye^z)\right)$

$+ k\left(\frac{d}{dx}(xe^z) - \frac{d}{dy}(ye^z)\right)$

$= i(xe^z - xe^z) - j(ye^z - ye^z) + k(e^z - e^z)$

$= \langle 0, 0, 0 \rangle = 0$

F is conservative. < we can use the fundamental theorem of line integrals to calculate this integrals.

$1 \le t \le 2$

$r(1) = (1, 1, 0)$

$r(2) = (4, 8, 1)$

$f(4,8,1) - f(1,1,0) = 4 \times 8 \times e - 1 \times 1 \times e^0$

$= 32e - 1$

$\boxed{\text{Ans: } 32e - 1}$

9. Find a potential function for F or determine that F is not conservative.

$F = y^2 i + (2xy + e^z) j + ye^z k$.

$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ y^2 & 2xy+e^z & ye^z \end{vmatrix}$$

$i\left(\frac{d}{dy}(ye^z) - \frac{d}{dz}(2xy+e^z)\right) - j\left(\frac{d}{dx}(ye^z) - \frac{d}{dz}(y^2)\right)$
$+ k\left(\frac{d}{dx}(2xy+e^z) - \frac{d}{dy}(y^2)\right)$

$= i(e^z - e^z) - j(\cancel{0-0}\ 0-0) + k(2y - 2y)$

$= 0$.

F is conservative.

$f\_x = y^2$ $\quad f\_y = 2xy + e^z$ $\quad f\_z = ye^z$.

$f = y^2 x + g(y,z)$

$\cancel{f = \frac{1}{3}y^3 + g(y,z)}$

$f\_y = 2y\cancel{x} + g'(y) = 2xy + e^z$

$g'(y) = e^z$

$g(y) = e^z y$.

$f = y^2 x + e^z y + h(z)$

$f\_z = ye^z + h'(z) = ye^z$

$h'(z) = 0$

$h(z) = 0$.

$f = y^2 x + e^z y + 0$

$\boxed{f = y^2 x + e^z y}$

final answer.

13. $F = \langle z\sec^2 x, z, y+\tan x \rangle$ find a potential function for F or determine that F is not conservative.

$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ z\sec^2 x & z & y+\tan x \end{vmatrix}$$

$i\left(\frac{d}{dy}(y+\tan x) - \frac{d}{dz}(z)\right) - j\left(\frac{d}{dx}(y+\tan x) - \frac{d}{dz}(z\sec^2 x)\right)$

$+ k\left(\frac{d}{dx}(z) - \frac{d}{dy}(z\sec^2 x)\right)$

$= i(1-1) - j(\sec^2 x - \sec^2 x) + k\cdot 0.$

$= 0$.

F is conservative.

$f\_x = z\sec^2 x \quad f\_y = z, \quad f\_z = y + \tan x.$

$f = z\tan x + g(y,z)$

$f\_y = g'(y) = z$

$g(y) = zy$

$f = z\tan x + zy + h(z)$

$f\_z = \tan x + y + h'(z) = y + \tan x.$

$h'(z) = 0$

$h(z) = 0$

$$\boxed{f = z\tan x + zy}$$

15. $F = \langle 2xy+5,\ x^2-4z,\ -4y \rangle$ Find a potential function for F or determine that F is not conservative.

$$\begin{array}{ccc} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 2xy+5 & x^2-4z & -4y \end{array}$$

$$i\left(\frac{d}{dy}(-4y) - \frac{d}{dz}(x^2-4z)\right) - j\left(\frac{d}{dx}(-4y) - \frac{d}{dz}(2xy+5)\right)$$

$$+ k\left(\frac{d}{dx}(x^2-4z) - \frac{d}{dy}(2xy+5)\right)$$

$$= i(-4+4) - j(0-0) + k(2x-2x)$$

$$= 0.$$

$f\_x = 2xy+5 \quad f\_y = x^2-4z \quad f\_z = -4y.$

$f = x^2y+5x+g(y,z)$

$f\_y = x^2+g'(y) = x^2-4z$

$g'(y) = -4z$

$g(y) = -4zy$

$f = x^2y+5x-4zy+h(z)$

$f\_z = -4y+h'(z) = -4y$

$h'(z)=0$

$h(z)=0.$

$f = x^2y+5x-4zy+0.$

$$\boxed{f = x^2y+5x-4zy.}$$

final ans

?17. evaluate

$$\int_C 2xyz\,dx + x^2z\,dy + x^2y\,dz$$

over the path $r(t) = (t^2, \sin(\pi t/4), e^{t^2-2t})$ for $0 \le t \le 2$.

$r(2) = (4, 1, 1)$

$r(0) = (0, 0, 1)$.

$$\cancel{f}\quad x^2yz + x^2zy + x^2yz \Big|_{(0,0,1)}^{(4,1,1)}$$

$= (16 + 16 + 16) - 0$.

$= \cancel{48}$. 16.

$\boxed{\text{Ans: } 48}$.

7.19. $f = x^2y - z$, $r_1 = \langle t, t, 0\rangle$ for $0 \le t \le 1$, and $r_2 = \langle t, t^2, 0\rangle$ for $0 \le t \le 1$

~~Let~~ $r_1(0) = \langle 0,0,0\rangle$ $r_1(1) = \langle 1,1,0\rangle$

$r_2(0) = \langle 0,0,0\rangle$ $r_2(1) = \langle 1,1,0\rangle$

$f(r_1(1)) - f(r_1(0)) = 1 - 0 = 1$

$f(r_2(1)) - f(r_2(0)) = 1 - 0 = 1$

$1 = 1$

\# Ans: they are same.