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16.2 $\longrightarrow$ 3, 9, 11, 13, 17, 27, 29, 31, 35

16.3 $\longrightarrow$ 1, 3, 5, 9, 13, 15, 17, 19

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16.2

(3) $F = \langle y^2, x^2 \rangle$ $\quad C \to y = x^{-1} \quad 1 \le x \le 2$

$dr = r'(t)\,dt$

$r(t) = (t, t^{-1})$

$F(r(t)) \to = (x^{-2}, x^2)$

a. $\boxed{F(r(t)) = \langle t^{-2}, t^2 \rangle}$

$\boxed{dr = \langle 1, -t^{-2} \rangle dt}$

b.

dot product $= t^{-2} + -1 = t^{-2} - 1$

* $F(r(t)) \cdot r'(t)\,dt = \int_C F \cdot dr$

$\int_1^2 (t^{-1} - 1)\,dt = \frac{1}{2}$ ?

(9) $f(x,y) = \sqrt{1+9xy}$ $\quad y = x^3$ for $0 \le x \le 1$

$y = y(t) \quad y'(t) = 3t^2$

$x = t$

$x'(t) = 1$

$\int_0^1 (\sqrt{1+9t^4})(\sqrt{9t^4+1})\,dt$

$\|r'(t)\| = \sqrt{9t^4+1}$

$= 2.8 = 2.8$ ✓

(11) $f(x,y,z) = z^2$ $\quad r(t) = (2t, 3t, 4t)$ for $0 \le t \le 2$

$r'(t) = (2, 3, 4)$

* exchange each variable with the original $r(t)$

$\int_0^2 (16t^2)\sqrt{29}\,dt$

$ds = \sqrt{4+9+16}$

$ds = \sqrt{29}\,dt$

$= 229.78$

$= 229.8$ ✓

(13) *** $f(x,y,z) = xe^{z^2}$ $\quad$ (0,0,1) to (0,2,0) to (1,1,1)

P Q R

PQ

$r(t) = (1-t)\langle 0,0,1 \rangle + t\langle 0,2,0 \rangle$

$r(t) = \langle 0, 0, 1-t \rangle + \langle 0, 2t, 0 \rangle$ $\quad r'(t) = \langle 0, 2, -1 \rangle$

$= \sqrt{3}$

$r(t) = \langle 0, 2t, 1-t \rangle \longrightarrow \int_0^1 0\,dt = 0$

* $\uparrow = 0!$

$r(t) = (1-t)\langle 0,2,0 \rangle + t\langle 1,1,1 \rangle$

$= \langle 0, 2-2t, 0 \rangle + \langle t, t, t \rangle = \langle t, 2-t, t \rangle$

$= \int_0^1 te^{t^2}\sqrt{3}\,dt = \frac{\sqrt{3}}{2}e^t \Big|_0^1 = \left(\frac{\sqrt{3}}{2}e - \frac{\sqrt{3}}{2} + 0\right)$

so far

add all 3 integrals together!

$r(t) = (1-t)\langle 1,1,1 \rangle + t\langle 0,0,1 \rangle$

$= \langle 1-t, 1-t, 1-t \rangle + \langle 0,0,t \rangle = \langle 1-t, 1-t, 1 \rangle$ $\quad r'(t) = \sqrt{3}$

$\int_0^1 (1-t)e\sqrt{3}\,dt = \frac{\sqrt{3}}{2}e \longrightarrow \frac{\sqrt{3}}{2}e + \frac{\sqrt{3}}{2}e - \frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{3}}{2}(e-1)}$

(17) $\int_C 1\,ds$ $\quad r(t) = (4t, -3t, 12t)$ $\quad 2 \le t \le 5$

$r'(t) = (4, -3, 12)$

$\|r'(t)\| = \sqrt{16+9+144} = \sqrt{169} = 13$

$\int_2^5 13\,dt$

$= 13t\Big|_2^5 = 13(5) - 13(2) = 65 - 26 = 39$

* This represents the distance between (8, -6, 24) and (20, -15, 60)

plug in $t = 2$ $\quad$ plug in $t = 5$

***

(27) $\int_C y\,dx - x\,dy$ $\quad$ P Q $\quad y = x^2$, $0 \le x \le 2$

$= \int y \cdot x'(t) - x \cdot y'(t)$

$= \int y(1) - x(2x)$

$= \int_0^2 y - 2x^2 = \int_0^2 (t^2 - 2t^2)\,dt$

$= \left[\frac{t^3}{3} - \frac{2t^3}{3}\right]\Big|_0^2 = \frac{8}{3} - \frac{16}{3} = -\frac{8}{3}$

$= -\frac{8}{3}$ ✓

(29) $\int_C (x-y)\,dx + (y-z)\,dy + z\,dz$

$(0,0,0) \longrightarrow (1,4,4)$

$r(t) = (1-t)\langle 0,0,0 \rangle + t\langle 1,4,4 \rangle$

$r(t) = \langle t, 4t, 4t \rangle$ $\quad r'(t) = \langle 1, 4, 4 \rangle$

exchange each variable with its value from $r(t)$

$\int_0^1 (t-4t)(1) + (4t-4t)(4)\,dy + (4t)(4)\,dt$

$= \int_0^1 ((1-3t) + 16t)\,dt = \int_0^1 (13t)\,dt$

$= \left[\frac{13}{2}t^2\right]\Big|_0^1 = \frac{13}{2} = \frac{13}{2}$ ✓

(31) $\int_C \frac{-y\,dx + x\,dy}{x^2+y^2}$ $\quad (1,0) \to (0,1)$

?

$r(t) = (1-t)(1,0) + t(0,1)$

$= (1-t, 0) + (0, t)$

$r(t) = (1-t, t)$

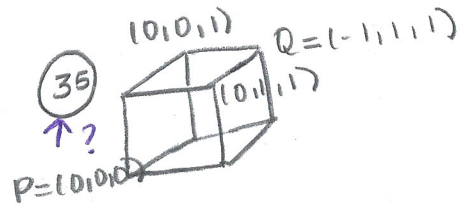
$r'(t) = (-1, 1)$

$x = \cos t$

$y = \sin t$

$\int_0^{2\pi} \frac{-t(-1) + (1-t)(1)}{\cos^2 t + \sin^2 t}$

(35) ?



16.3 *NOT the same as F(x, y, z)!! WATCH OUT! $\boxed{\int \nabla f \cdot dr = f(b) - f(a)}$ 11/15/20

① $f(x,y,z) = xy\sin(yz)$

$(0,0,0) \to (1,1,\pi)$ — a, b

$\int F \cdot dr = f(1,1,\pi) - f(0,0,0)$
$= \sin(\pi) - 0 = -1 - 0 = -1$

③ $F(x,y) = \langle 3, 6y\rangle$
$f(x,y) = 3x + 3y^2$
$r(t) = \langle t, 2t^{-1}\rangle$ for $1 \le t \le 4$

$f_x = 3 = 3$ ✓ $f(x,y) \Rightarrow \langle 3t + 12t^{-2}\rangle$
$f_y = 6y = 6y$ ✓

$\int F \cdot dr = f(4) - f(1)$
$= \langle 12 + 12(4^{-2})\rangle - \langle 3 + 12\rangle$
$= (12 + \frac{3}{4} - 15) = -3 + \frac{3}{4}$
$= -\frac{9}{4} = -\frac{9}{4}$ ✓
$\frac{3}{1} = -\frac{12}{4} + \frac{3}{4} = -\frac{9}{4}$

⑤ $F(x,y,z) = ye^z i + xe^z j + xye^z k$
$f(x,y,z) = xye^z$
$r(t) = (t^2, t^3, t-1)$ $1 \le t \le 2$
$f_x = ye^z = ye^z$ $f(t) = t^5 e^{t-1}$
$f_y = xe^z = xe^z$
$f_z = xye^z = xye^z$
$\int = f(2) - f(1) = 2^5 e^1 - 1$
$= (32e - 1) = (32e - 1)$ ✓

⑨ $F = y^2 i + (2xy + e^z) j + ye^z k$

$$\text{curl } F = \begin{vmatrix} i & j & k \\ dF/dx & dF/dy & dF/dz \\ y^2 & (2xy+e^z) & (ye^z) \end{vmatrix}$$

$= (e^z - e^z)i - (0-0)j + (2y - 2y)k = 0$ ✓
so conservative

$\int y^2 dx = \boxed{xy^2 + g(y,z)} \Rightarrow 2yx + g'(y,z) = 2xy + e^z$
$g'(y,z) = e^z$
$\int e^z dy = \boxed{ye^z + h(z)} \Rightarrow ye^z + h'(z) = ye^z \to h(z) = 0$
$\boxed{f(x,y,z) = xy^2 + ye^z}$

⑬ $F = \langle z\sec^2 x, z, y + \tan x\rangle$
$\int z\sec^2 x\, dx = z\tan x + g(y,z)$
$g'(y,z) = z$
$\int z\, dy = yz \longrightarrow y + h'(z) = y + \tan x$
$h(z) = \tan x$
$\boxed{f(x,y,z) = z\tan x + yz}$

⑮ $F = \langle 2xy + 5, x^2 - 4z, -4y\rangle$
$\int 2xy + 5 = x^2y + 5x + g(y,z)$
$x^2 + g'(y,z) = x^2 - 4z$
$\hookrightarrow = -4z$
$\int -4z\, dy = -4zy + h(z)$
$= -4y + h'(z) = -4y$
$h'(z) = 0$
$\boxed{f(x,y,z) = x^2y + 5x - 4zy}$

⑰ $\int_c 2xyz\,dx + x^2z\,dy + x^2y\,dz$
$r(t) = (t^2, \sin(\pi t/4), e^{t^2-2t})$
$0 \le t \le 2$
$= \int_0^2 (4t^4\sin(\pi t/4)e^{t^2-2t}) + t^4 e^{t^2-2t}(\pi/4)\cos(\pi t/4)$
$+ t^4\sin(\pi t/4)(2t-2)e^{t^2-2t}$
$= 16 = 16$ ✓

⑲ $f = x^2y - z$ $r_1 = \langle t, t, 0\rangle$
$0 \le t \le 1$
a. $f = t^2(t) - 0$
$= t^3 \longrightarrow = f(1) - f(0)$
$= 1$

b. $r(t) = \langle t, t^2, 0\rangle$ $0 \le t \le 1$
$f = t^3 - 0 = t^3$
$= f(1) - f(0)$
$= 1 - 0 = 1 = 1$
$1 = 1$ ✓