16.1 Homework

(3) $F = \langle y^2, x^2 \rangle$, $C \Rightarrow y = x^{-1}$ $x = [1,2]$

a) Get $F(r(t))$ and $dr = r'(t)\,dt$ $r(t) = \langle t, t^{-1} \rangle$

$F(r(t)) = \langle t^{-2}, t^2 \rangle$ $r'(t) = \langle 1, -t^{-2} \rangle$

$dr = \langle 1, -t^{-2} \rangle dt$

b) $F(r(t)) \cdot r'(t)\,dt$, evaluate $\int_C F \cdot dr$

$F(r(t)) \cdot r'(t)\,dt = \langle t^{-2}, t^2 \rangle \cdot \langle 1, -t^{-2} \rangle$

$= t^{-2} - 1$

$\int_1^2 (t^{-2} - 1)\,dt = -\frac{1}{2}$

(9) $f(x,y) = \sqrt{1+9xy}$, $y = x^3$, $0 \leq x \leq 1$

$\int_0^1 \sqrt{1+9x^4}\,dx = \frac{14}{5}$

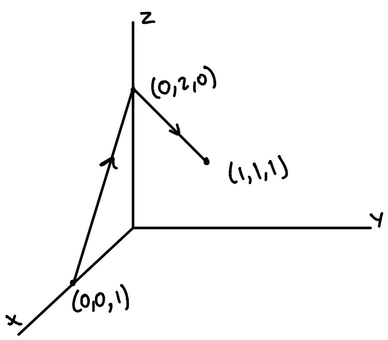
(11) $f(x,y,z) = z^2$, $r(t) = \langle 2t, 3t, 4t \rangle$, $0 \leq t \leq 2$

$r'(t) = \langle 2, 3, 4 \rangle$, $|r'(t)| = \sqrt{2^2+3^2+4^2} = \sqrt{29}$

$f(r(t)) = 16t^2$

$\int_0^2 (16t^2)(\sqrt{29})\,dt = \frac{128\sqrt{29}}{3}$

(13) $f(x,y,z) = xe^{z^2}$, Piecewise : $(0,0,1) \to (0,2,0) \to (1,1,1)$



$\int_C f(x,y,z) \cdot ds = \frac{\sqrt{3}}{2}(e-1)$

(17) $\int_C 1\, ds \qquad r(t) = \langle 4t, -3t, 12t \rangle \qquad 2 \le t \le 5$

$r'(t) = \langle 4, -3, 12 \rangle \to |r'(t)| = \sqrt{16+9+144}$

$\int_2^5 (1)(13)dt = 39$

integral represents distance from $(4(2), -3(2), 12(2))$ to $(4(5), -3(5), 12(5))$.

(27) $\int_C y\,dx - x\,dy \quad , \quad y = x^2 \quad , \quad 0 \le x \le 2$

$dy = 2x\,dx$

$\int_0^2 x^2 dx - (x)(2x)dx = \left[\frac{x^3}{3} - \frac{2x^3}{3} \right]_0^2 = \left[-\frac{x^3}{3} \right]_0^2$

$= -\frac{8}{3}$

(29) $\int_C (x-y)\,dx + (y-z)\,dy + z\,dz$, line from $(0,0,0)$ to $(1,4,4)$

$r(t) = \langle t, 4t, 4t \rangle$

$r'(t) = \langle 1, 4, 4 \rangle \to |r'(t)| = \sqrt{1^2+4^2+4^2} = \sqrt{33}$

$F(r(t)) = \langle x-y, y-z, z \rangle = \langle t-4t, 4t-4t, 4t \rangle$

$F(r(t)) = \langle -3t, 0, 4t \rangle$

$F(r(t)) \cdot r'(t) = \langle -3t, 0, 4t \rangle \cdot \langle 1, 4, 4 \rangle$

$= -3t + 16t$

$F(r(t)) = 13t$

$\int_0^1 13t\,dt = \left[\frac{13t^2}{2}\right]_0^1 = \frac{13}{2}$

31 $\displaystyle\int_C \frac{-y\,dx + x\,dy}{x^2 + y^2}$, line from $(1,0)$ to $(0,1)$

$r(t) = (1-t)\langle 1,0 \rangle + t\langle 0,1 \rangle$

$r(t) = \langle 1-t, t \rangle$

$r'(t) = \langle 1, 1 \rangle \Rightarrow |r'(t)| = \sqrt{1^2+1^2} = \sqrt{2}$

$F(r(t)) = \left\langle -\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle = \left\langle -\frac{t}{(1-t)^2+t^2}, \frac{1-t}{(1-t)^2+t^2} \right\rangle$

$$\int_0^1 \left(\frac{t}{(1-t)^2+t^2} + \frac{1-t}{(1-t)^2+t^2} \right) dt = \frac{\pi}{2}$$

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$C_1: (0,0,0) \to (0,0,1) \Rightarrow (1-t)\langle 0,0,0 \rangle + t\langle 0,0,1 \rangle = \langle 0,0,t \rangle$

$C_2: (0,0,1) \to (0,1,1) \Rightarrow (1-t)\langle 0,0,1 \rangle + t\langle 0,1,1 \rangle = \langle 0,t,1 \rangle$

$C_3: (0,1,1) \to (-1,1,1) \Rightarrow (1-t)\langle 0,1,1 \rangle + t\langle -1,1,1 \rangle = \langle -t,1,1 \rangle$

$C_1'(t) = \langle 0,0,1 \rangle$, $C_2'(t) = \langle 0,1,0 \rangle$, $C_3'(t) = \langle -1,0,0 \rangle$

$$\int_C F \cdot ds = \int_{C_1} F \cdot ds + \int_{C_2} F \cdot ds + \int_{C_3} F \cdot ds$$

$$\int_0^1 F(r(t))\, r'(t)\, dt = \int_0^1 (0 + 0 + 1)\, dt = \left[t\right]_0^1 = 1$$

$$\int_0^1 F(r(t))\, r'(t)\, dt = \int_0^1 (0 + e^{-t} + 0)\, dt = 1 - \frac{1}{e}$$

$$\int_0^1 F(r(t))\, r'(t)\, dt = \int_0^1 (e + 0 + 0)\, dt = \left[te\right]_0^1 = e$$

$$\int_C F \cdot ds = 1 + 1 - \frac{1}{e} + e$$

$$\int_C F \cdot ds = 2 - \frac{1}{e} + e$$

16.3 Homework

① $f(x,y,z) = xy\sin(yz)$

$\nabla f = \langle y\sin(yz),\ x(\sin(yz) + yz\cos(yz)),\ xy^2\cos(yz) \rangle$

$F = y\sin(yz)\,i + x(\sin(yz) + yz\cos(yz))\,j + xy^2\cos(yz)\,k$

Start: $f(0,0,0) = 0$, end: $f(1,1,\pi) = 0$

$\int_0^0 y\sin(yz)\,dx + x(\sin(yz) + yz\cos(yz))\,dy + xy^2\cos(yz)\,dz$

$\int_c F \cdot dr = 0$

③ $F(x,y) = \langle 3, 6y \rangle$, $f(x,y) = 3x + 3y^2$, $r(t) = \langle t, 2t^{-1} \rangle$, $1 \le t \le 4$

$r'(t) = \langle 1, -2t^{-2} \rangle$

$F(r(t)) = \langle 3, \frac{12}{t} \rangle$

$\int F(r(t)) \cdot r'(t)\,dt = \int_1^4 \langle 3, \frac{12}{t} \rangle \cdot \langle 1, -2t^{-2} \rangle\,dt$

$\int_1^4 (3 - \frac{24}{t^3})\,dt = 12 + \frac{12}{16} - (3+12)$

$= -\frac{9}{4}$

⑤ $F(x,y,z) = ye^z i + xe^z j + xye^z k$, $f(x,y,z) = xye^z$, $f(x,y,z) = xye^z$

$r(t) = \langle t^2, t^3, t-1 \rangle$ for $1 \le t \le 2$

$r'(t) = \langle 2t, 3t^2, 1 \rangle$ $\qquad F(r(t)) = (3t^2)e\,i + (2t)e\,j + (6t^3)e\,k$

$\int F(r(t)) \cdot r'(t)\,dt = \int \langle 3t^2 e, 2te, 6t^3 e \rangle \cdot \langle 2t, 3t^2, 1 \rangle\,dt$

$= \int 6t^3 e + 6t^2 e + 6t^3 e\,dt = \int 18t^3 e\,dt$

$= \left[\frac{9t^5 e}{2}\right]$

$= \frac{135e}{2}$

(9) $F = y^2 i + (2xy + e^z) j + ye^z k$

$\text{curl}(F) = \langle 0, 0, 0 \rangle$ F is conservative

$f_x = y^2$, $f_y = (2xy + e^z)$, $f_z = ye^z$

$f = \int f_x dx = xy^2 + h(y,z)$

$f_y = 2xy + h_y$

$f = xy^2 + ye^z + g(z)$

$f_z = y e^z$

$f = xy^2 + ye^z + g(z)$, $g(z) = 0$

$f = xy^2 + ye^z$

(13) $F = \langle z\sec^2 x, z, y + \tan x \rangle$

$\text{curl } F = \langle 0, 0, 0 \rangle$. F is conservative

$f_x = z\sec^2 x$, $f_y = z$, $f_z = y + \tan x$

$f = \int f_x dx = z\tan x + h(y,z)$

$f_y = h_y$, $h_y = z$

$f = z\tan x + yz + g(z)$

$f = z\tan x + yz$

(15) $F = \langle 2xy + 5, x^2 - 4z, -4y \rangle$

$f_x = 2xy + 5$, $f_y = x^2 - 4z$, $f_z = -4y$

$f = \int f_x dx = x^2 y + 5x + h(y,z)$

$f_y = x^2 + h_y$, $h_y = -4z$

$f = x^2 - 4yz + g(z)$

$f = x^2 - 4yz$

17 $\int_C 2xyz\,dx + x^2z\,dy + x^2y\,dz$, $r(t) = \langle t^2, \sin(\pi t/4), e^{t^2-2t} \rangle$

$r'(t) = \langle 2t, \frac{\pi\cos(\frac{\pi t}{4})}{4}, (2t-2)e^{t^2-2t} \rangle$

$F(r(t)) = 2(t^2)(\sin(\frac{\pi t}{4}))(e^{t^2-2t})\,i + (t^4)(e^{t^2-2t})\,j + (t^4)(\sin(\frac{\pi t}{4}))\,k$

$\int_0^2 F(r(t)) \cdot r'(t)\,dt = 16$