16.2: 3 9 11 13 17 27 29 31 35
16.3: 1 3 5 9 13 15 17 19

Chapter 16 HW

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16.2

3) $F = \langle y^2, x^2 \rangle \quad y = \frac{1}{x} \quad 1 \le x \le 2$

a) $r(t) = \langle t, t^{-1} \rangle \quad r'(t)dt = dr = \langle 1, \frac{-1}{t^2} \rangle$

$F(r(t)) = \frac{1}{t^2}\hat{i} + t^2\hat{j}$

b) $\int F(r(t)) \cdot dr = \int_1^2 \frac{1}{t^2} \cdot 1\,dt + t^2 \cdot \left(\frac{-1}{t^2}\right)dt$

$= \left(\frac{-1}{t^3} - t\right)\Big|_1^2 = \left(\frac{-1}{8} - 2\right) - (-1 - 1) = \boxed{-\frac{1}{8}}$

Ans in Book wrong?

9) $f(x,y) = \sqrt{1 + 9xy} \quad y = x^3 \quad y(t) = (x(t))^3 \Rightarrow \boxed{\begin{matrix} y = t^3 \\ x = t \end{matrix}}$

$r(t) = \langle t, t^3 \rangle \quad r'(t) = \langle 1, 3t^2 \rangle$

$0 \le t \le 1$

$f(x(t), y(t)) = \sqrt{1 + 9t^4}$

$\int_C f\,ds = \int_0^1 \sqrt{1 + 9t^4}\,\sqrt{(1)^2 + (3t^2)^2}\,dt$

$= \int_0^1 \sqrt{1 + 9t^4}\,\sqrt{1 + 9t^4}\,dt = \int_0^1 1 + 9t^4\,dt$

$= \left(t + \frac{9t^5}{5}\right)\Big|_0^1 = \boxed{\frac{14}{5}}$

16.2 Cont

11) $f(x,y,z)=z^2 \quad r(t)=\langle 2t, 3t, 4t\rangle \quad 0\le t\le 2$

$$r'(t)=\langle 2, 3, 4\rangle$$

$$|r'(t)dt| = \sqrt{2^2+3^2+4^2} = \sqrt{2^2+5^2} = \sqrt{29}$$

$$\int_C f\,ds = \int_0^2 16t^2\cdot\sqrt{29}\,dt = \frac{16\sqrt{29}}{3}\cdot 8 = \boxed{\frac{128\sqrt{29}}{3}}$$

13) $f(x,y,z)=xe^{z^2}$ First path: $v(t)=(0,0,1)+\boxed{t\langle 0,2,-1\rangle}$

2nd " : " $=(0,2,0)+\boxed{t\langle 1,-1,1\rangle}$

$r_1(t)=\langle 0, 2t, 1-t\rangle$

$r_2(t)=\langle t, 2-t, t\rangle;\ r_2'(t)=\langle 1,-1,1\rangle$

$|r_2'(t)|=\sqrt{3}$

$$\int_0^1 0\cdot e^{(1-t)^2}\sqrt{2^2+(-1)^2}\,dt = \boxed{1}$$

$$+\int_0^1 t\cdot e^{t^2}\sqrt{3}\,dt = \int_0^1 \frac{\sqrt{3}}{2}e^u\,du = \boxed{\frac{\sqrt{3}}{2}(e-1)}$$

$u=t^2\big|_0^1 = \big|_0^1$

$du=2t$

$\underline{16.2 \text{ Cont}}$

17) $\int 1\,ds \quad C: r(t) = (4t, -3t, 12t) \quad 2 \le t \le 5$

$|r'(t)dt| = 13$

$= \int_2^5 13\,dt = 13(3) = \boxed{39}$

$\boxed{\text{Int represents dist. btwn } r(5) \text{ \& } r(2)}$

27) $\int y\,dx + (-x)\,dy \quad y = x^2 \quad 0 \le x \le 2$

$x = t$	$y = t^2$	$0 \le t \le 2$
$x'(t) = 1$	$y'(t) = 2t$	

$= \int_0^2 \left((t^2)(1) - (t)(2t)\right) dt = t^3\left(\frac{1}{3} - \frac{2}{3}\right)$

$= \left(\frac{-t^3}{3}\right)\Big|_0^2 = \boxed{\frac{-8}{3}}$

29) $\int_C (x-y)\,dx + (y-z)\,dy + z\,dz \quad r(t) = \langle t, 4t, 4t \rangle \quad 0 \le t \le 1$

$r'(t)dt = \langle 1, 4, 4 \rangle$

$= \int_0^1 \left((-3t)(1) + (0)(4) + (4t)(4)\right) dt$

$= \left(\frac{13}{2}t^2\right)\Big|_0^1 = \boxed{\frac{13}{2}}$

16.2 Cont

31) $\int \frac{-y\,dx + x\,dy}{x^2+y^2}$ $\quad r(t) = (1,0) + t\langle -1, +1\rangle \quad 0 \le t \le 1$

$x(t) = 1-t \quad y(t) = t$

$x'(t) = -1 \quad y'(t) = 1$

$$= \int_0^1 \frac{1}{(1-2t+2t^2)} (-t)(-1) + (1-t) \; dt$$

$$\downarrow$$

$$\frac{2 \pm \sqrt{4-8}}{4} = \frac{1}{2}$$

Done in Maple

$$= \int_0^1 \frac{1}{1-2t+2t^2}\, dt = \boxed{\frac{\pi}{2}}$$

(I'm assuming this is some inverse trig weirdness)

35) $F(x,y,z) = \langle e^z, e^{x-y}, e^y\rangle$ 3 paths, piecewise

$$\begin{cases} x_1(t)=0, & y_1(t)=0, & z_1(t)=t \\ x_2(t)=0, & y_2(t)=t, & z_2(t)=1 \\ x_3(t)=-t & y_3(t)=1, & z_3(t)=1 \end{cases}$$

$$\int_0^1 \cancel{(e^t)(0)} + \cancel{(e^0)(0)} + (e^0)(1)\,dt$$

$$= 1 + \int_0^1 \cancel{(e^1)(0)} + (e^{0-t})(1) + 0\,dt$$

$$= 1 + (e\cdot t - e^{-t})\Big|_0^1 = 1 + \left(\left(e - \frac{1}{e}\right) - \left(0 - \frac{1}{1}\right)\right)$$

$$+ \int_0^1 (e^1)(-1) + \cancel{(e^{t-1})(0)} + \cancel{(e)(0)}\,dt$$

16.2 Cont

35) Ans =

$$2 + e - \frac{1}{e} + \int_0^1 -e\,dt =$$

$$\boxed{2 - e - \frac{1}{e}}$$

16.3 HW

1) $f(x, y, z) = xy\sin(yz)$ $\quad F = \nabla f$ $\quad$ C: any path from $a = (0, 0, 0)$ to $b = (1, 1, \pi)$

$$\int_C F \cdot dr = f(b) - f(a) = 0 - 0 = \boxed{0}$$

3) $F(x, y) = \langle 3, 6y \rangle$ $\quad f(x, y) = 3x + 3y^2$

$\nabla f = F = \langle 3, 6y \rangle$ $\quad r(t) = \langle t, \frac{2}{t} \rangle$ $\quad 1 \le t \le 4$

$$f(x(t), y(t)) = 3t + \frac{12}{t^2}$$

$$f(x(4), y(4)) - f(x(1), y(1)) = \left(12 + \frac{3}{4}\right) - (15) = \boxed{-\frac{9}{4}}$$

5) $F(x, y, z) = ye^z\hat{i} + xe^z\hat{j} + xye^z\hat{k}$ $\quad f(x,y,z) = xye^z$

$\nabla f = F = \langle ye^z, xe^z, xye^z \rangle$ $\quad r(t) = (t^2, t^3, t-1)$ $\quad 1 \le t \le 2$

Continued next pg

16.3 Cont

5) $f(x(t), y(t), z(t)) = (t^2)(t^3)e^{t-1} = \frac{t^5 e^t}{e}$

$f(a) - f(b) = \frac{(2)^5 e^{\cancel{2}}}{\cancel{e}} - \frac{(1)^5 \cancel{e^1}}{\cancel{e}} = 32e - 1$

9) $F = y^2\hat{\imath} + (2xy + e^z)\hat{\jmath} + ye^z\hat{k}$.

$$\boxed{f = xy^2 + ye^z}$$

Check Conservative:

$$\det \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & 2xy+e^z & ye^z \end{vmatrix}$$

$= (e^z - e^z)\hat{\imath} - (0-0)\hat{\jmath} + (2y - 2y)\hat{k}$

$= 0$ ✓

13) $F = \langle z\sec^2 x,\ z,\ y + \tan x \rangle$

$$\boxed{f = yz + z\tan x}$$

Check

$$\det \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z\sec^2 x & z & y+\tan x \end{vmatrix}$$

$= (1-1)\hat{\imath} - (\sec^2 x - \sec^2 x)\hat{\jmath} + (0-0)\hat{k}$

$= 0$ ✓

16.3 HW

15) $F = \langle 2xy+5,\ x^2-4z,\ -4y \rangle$

$$\boxed{f = x^2y + 5x - 4yz}$$

<u>Check</u>

$$\det \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy+5 & x^2-4z & -4y \end{vmatrix}$$

$$= (-4-(-4))\hat{\imath} - (0-0)\hat{\jmath} + (2x-2x)\hat{k}$$

$$= 0 \checkmark$$

17) $$\int_C 2xyz\,dx + x^2z\,dy + x^2y\,dz$$ $C: r(t) = (t^2, \sin(\frac{\pi t}{4}), e^{t^2-2t})$, $0 \le t \le 2$

$$= \int F\,dr \quad F = \nabla f \quad \boxed{f = x^2yz}$$

$f(x(2), y(2), z(2)) - f(x(0), y(0), z(0)) =$

$((2)^2 \cdot \sin(\frac{\pi}{2}) \cdot e^0) - (0) = \boxed{2}$

<u>Check</u>

$$\det \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz & x^2z & x^2y \end{vmatrix}$$

$$= (x^2-x^2)\hat{\imath} - (2xy-2xy)\hat{\jmath} + (2xz-2xz)\hat{k}$$

$$= 0 \checkmark$$

16.3 Cont

19) $f = x^2 y - z$ $\qquad r_1 = \langle t, t, 0 \rangle$ $\qquad 0 \le t \le 1$

$r_2 = \langle t, t^2, 0 \rangle$

$F = \nabla f = \langle 2xy, x^2, -1 \rangle$

$r_1' = \langle 1, 1, 0 \rangle$

$f(x(1), y(1), z(1)) - f(x(0), y(0), z(0)) = (1) - 0 = \boxed{1}$

$r_2' = \langle 1, 2t, 0 \rangle$

$$\int 2xy\,dx + x^2 dy - 1\,dz \overset{?}{=} \int 2xy\,dx + x^2 dy - 1\,dz$$

$$\int 2(t)(t)(dt) + t^2 dt + \cancel{0} \overset{?}{=} \int 2(t)(t^2)dt + t^2(2t)dt + \cancel{0}$$

$$\int 3t^2\,dt \overset{?}{=} \int 4t^3$$

$$(t^3)\Big|_0^1 \overset{?}{=} (t^4)\Big|_0^1 = \boxed{1} \checkmark$$